8.2.xa. Exercise answers

1. **a.** *Everyone has seen a bear*
   Everyone is such that (he or she has seen a bear)
   (∀x: x *is a person*) x *has seen a bear*
   (∀x: Px) *a bear is such that (x has seen it)*
   (∀x: Px) (∃y: y *is a bear*) x *has seen* y

   $$(∀x: Px) (∃y: By) Sxy$$
   $$∀x (Px → ∃y (By ∧ Sxy))$$

   [B: λx (x *is a bear*); P: λx (x *is a person*); S: λxy (x *has seen* y)]

   b. *Everyone was talking about a certain movie*
   A certain movie is such that (everyone was talking about it)
   (∃x: x *is a movie*) *everyone was talking about* x
   (∃x: Mx) *everyone is such that (he or she was talking about* x)
   (∃x: Mx) (∀y: y *is a person*) y *was talking about* x

   $$(∃x: Mx) (∀y: Py) Tyx$$
   $$∃x (Mx ∧ ∀y (Py → Tyx))$$

   [M: λx (x *is a movie*); P: λx (x *is a person*); T: λxy (x *was talking about* y)]

   c. *A capital was chosen by each state*
   Each state is such that (a capital was chosen by it)
   (∀x: x *is a state*) *a capital was chosen by* x
   (∀x: Sx) *a capital is such that (it was chosen by* x)
   (∀x: Sx) (∃y: y *is a capital*) y *was chosen by* x

   $$(∀x: Sx) (∃y: Cy) Hyx$$
   $$∀x (Sx → ∃y (Cy ∧ Hyx))$$

   [C: λx (x *is a capital*); H: λxy (x *was chosen by* y); S: λx (x *is a state*)]

d. *There is a capital that was chosen by each state*
Something is a capital that was chosen by each state
Something is such that (it is a capital that was chosen by each
    state)
∃x x *is a capital that was chosen by each state*
∃x (x *is a capital* ∧ x *was chosen by each state*)
∃x (Cx ∧ *each state is such that (x was chosen by it)*)
∃x (Cx ∧ (∀y: y *is a state*) x *was chosen by* y)

$$\exists x \, (Cx \land (\forall y: Sy) \, Hxy)$$
$$\exists x \, (Cx \land \forall y \, (Sy \rightarrow Hxy))$$

[C: λx (x *is a capital*); H: λxy (x *was chosen by* y); S: λx (x *is a state*)]

e. *Someone who no reporter knew leaked the information*
Someone who no reporter knew is such that (he or she leaked
    the information)
(∃x: x *is a person who no reporter knew*) x *leaked the*
    information
(∃x: x *is a person* ∧ *no reporter knew* x) Lxi
(∃x: Px ∧ *no reporter is such that (he or she knew* x*)*) Lxi
(∃x: Px ∧ (∀y: y *is a reporter*) ¬ y *knew* x) Lxi

$$(\exists x: Px \land (\forall y: Ry) \neg Kyx) \, Lxi$$
$$\exists x \, ((Px \land \forall y \, (Ry \rightarrow \neg Kyx)) \land Lxi)$$
$$or: (\exists x: Px \land \neg (\exists y: Ry) \, Kyx) \, Lxi$$

[K: λxy (x knew y); L: λx (x *leaked* y); P: λx (x *is a person*); R: λx (x *is a reporter*); i: *the information*]

f. *A head of a horse is the head of a mammal*
Every head of a horse is such that (it is the head of a mammal)
(∀x: x *is the head of a horse*) x *is the head of a mammal*
(∀x: *a horse is such that (x is the head of it)*) *a mammal is such*
    that (x is the head of it)
(∀x: (∃y: y *is a horse*) x *is the head of* y) (∃z: z *is a mammal*) x
    is the head of z
(∀x: (∃y:Hy) x = *the head of* y) (∃z: Mz) x = *the head of* z

$$(\forall x: (\exists y: Hy) \, x = hy) \, (\exists z: Mz) \, x = hz$$
$$\forall x \, (\exists y \, (Hy \land x = hy) \rightarrow \exists z \, (Mz \land x = hz))$$
$$or: (\forall x: (\exists y: Hy) \, Dxy) \, (\exists z: Mz) \, Dxz$$

[D: λxy (x *is a head of* y); H: λx (x *is a horse*); M: λx (x *is a mammal*); h: λx (*the head of* x)]

In this interpretation, which seems most natural given the content of the sentence, *a* is understood to indicate a generalization rather than a claim of exemplification. That is, it amounts to *any* in a use that is equivalent to *every* rather than in contrast with it. It appears in a location where *any* would not contrast with *every*, so if the sentence were understood to make a claim of exemplification, substituting *any* for *a* would change the meaning.

g. *Everyone who has seen a rainbow has seen a rainstorm*
Everyone who has seen a rainbow is such that (he or she has seen a rainstorm)
(∀x: x *is person who has seen a rainbow*) x *has seen a rainstorm*
(∀x: x *is person* ∧ x *has seen a rainbow*) *a rainstorm is such that (*x *has seen it)*
(∀x: x *is person* ∧ *a rainbow is such that (*x *has seen it))* (∃z: z *is a rainstorm*) x *has seen* z
(∀x: x *is person* ∧ (∃y: y *is a rainbow*) x *has seen* y) (∃z: Rz) Sxz

$$\text{(∀x: Px ∧ (∃y: Ry) Sxy) (∃z: Tz) Sxz}$$
$$\forall x\ (\ (Px \wedge \exists y\ (Ry \wedge Sxy)) \rightarrow \exists z\ (Tz \wedge Sxz)\ )$$

[P: λx (x *is a person*); R: λx (x *is a rainbow*); S: λxy (x *has seen* y); T: λx (x *is a rainstorm*)]

h. *Every child was given a toy by each Santa*
Every child is such that (he or she was given a toy by each Santa)
(∀x: x *is a child*) x *was given a toy by each Santa*
(∀x: Cx) *each Santa is such that (*x *was given a toy by him or her)*
(∀x: Cx) (∀y: y *is a Santa*) x *was given a toy by* y
(∀x: Cx) (∀y: Sy) *a toy is such that (*x *was given it by* y)*
(∀x: Cx) (∀y: Sy) (∃z: z *is a toy*) x *was given* z *by* y

$$\text{(∀x: Cx) (∀y: Sy) (∃z: Tz) Gxzy}$$
$$\forall x\ (Cx \rightarrow \forall y\ (Sy \rightarrow \exists z\ (Tz \wedge Gxzy)))$$

[C: λx (x *is a child*); G: λxyz (x *was given* y *by* z); S: λx (x *is a Santa*); T: λx (x *is a toy*)]

Notice that, in spite of the capitalization, *Santa* is not used here as a proper name but instead as a sort of job title. As a result it is represented not by an individual term but instead by a predicate. For representation by an individual term to be appropriate, it would have to be possible to paraphrase the sentence using *each thing that is Santa* rather than *each thing that is a Santa*.

i. *There is a toy that every child was given by each Santa*
Something is a toy that every child was given by each Santa
∃x x *is a toy that every child was given by each Santa*
∃x (x *is a toy* ∧ *every child was given* x *by each Santa*)
∃x (Tx ∧ *every child is such that (he or she was given* x *by each Santa))*
∃x (Tx ∧ (∀y: y *is a child*) y *was given* x *by each Santa*)
∃x (Tx ∧ (∀y: Cy) *each Santa is such that (y was given* x *by him or her))*
∃x (Tx ∧ (∀y: Cy) (∀z: z *is a Santa*) y *was given* x *by* z)

$$∃x (Tx ∧ (∀y: Cy) (∀z: Sz) Gyxz)$$
$$∃x (Tx ∧ ∀y (Cy → ∀z (Sz → Gyxz)))$$

[C: λx (x *is a child*); G: λxyz (x *was given* y *by* z); S: λx (x *is a Santa*); T: λx (x *is a toy*)]

2. **a.** ∀x ∃y x *depends on* y
∀x *something is such that (x depends on it)*
∀x x *depends on something*
Everything is such that (it depends on something)

Everything depends on something

b. ∃x ∀y x *depends on* y
∃x *everything is such that (x depends on it)*
∃x x *depends on everything*
Something is such that (it depends on everything)

Something depends on everything

c. ∀x ∃y y *depends on* x
∀x *something is such that (it depends on* x*)*
∀x *something depends on* x

Everything is such that something depends on it
or: Everything has something depending on it
or (perhaps): Something or other depends on each thing

d. ∃x ∀y y *depends on* x
∃x *everything is such that (it depends on* x*)*
∃x *everything depends on* x

Something is such that everything depends on it
or: Something has everything depending on it
or: There is something that everything depends on
or (perhaps): All things depend on a certain thing

e. ($\forall$x: x *is a person* $\wedge$ x *is humble*) ($\exists$y: y *is a person*) x *admires* y
($\forall$x: x *is a humble person*) *someone is such that (*x *admires him or her)*
($\forall$x: x *is a humble person*) x *admires someone*
Every humble person is such that (he or she admires someone)

Every humble person admires someone
or: Everyone who is humble admires someone

f. ($\exists$y: y *is a person*) ($\forall$x: x *is a person* $\wedge$ x *is humble*) x *admires* y
($\exists$y: y *is a person*) ($\forall$x: x *is a humble person*) x *admires* y
($\exists$y: y *is a person*) *every humble person is such that (he or she admires* y*)*
($\exists$y: y *is a person*) *every humble person admires* y

Someone is such that every humble person [*or: everyone who is humble*] *admires him or her*
or: Someone has every humble person admiring him or her
or: There is someone [*or: a person*] *who every humble person admires*
or (perhaps): All who are humble admire a certain person

g. $\neg$ ($\forall$x: x *is a person* $\wedge$ ($\exists$y: y *is a person*) x *admires* y) x *is humble*
$\neg$ ($\forall$x: x *is a person* $\wedge$ *someone is such that (*x *admires him or her)*) x *is humble*
$\neg$ ($\forall$x: x *is a person* $\wedge$ x *admires someone*) x *is humble*
$\neg$ ($\forall$x: x *is a person who admires someone*) x *is humble*
$\neg$ *everyone who admires someone is such that (he or she is humble)*
$\neg$ *everyone who admires someone is humble*

Not everyone who admires someone is humble
or: Not everyone who admires anyone is humble

h.   ¬ (∃x: x *is a person*) (∀y: y *is a person* ∧ y *has seen* x) x *has seen* y

¬ (∃x: x *is a person*) (∀y: y *is a person who has seen* x) x *has seen* y

¬ (∃x: x *is a person*) *everyone who has seen* x *is such that (*x *has seen him or her)*

¬ (∃x: x *is a person*) x *has seen everyone who has seen* x

¬ *someone is such that (he or she has seen everyone who has seen him or her)*

¬ *someone has seen everyone who has seen him or her*

No one has seen everyone who has seen him or her

i.   ¬ (∃x: x *is a person* ∧ (∀y: ¬ (y *is a person* ∧ y *has spoken to* x)) ¬ x *has spoken to* y) x *is an extrovert*

¬ (∃x: x *is a person* ∧ (∀y: ¬ y *is a person who has spoken to* x) ¬ x *has spoken to* y) x *is an extrovert*

¬ (∃x: x *is a person* ∧ *only people who have spoken to* x *are such that(*x *has spoken to them))*) x *is an extrovert*

¬ (∃x: x *is a person* ∧ x *has spoken only to people who have spoken to* x) x *is an extrovert*

¬ (∃x: x *is a person who has spoken only to people who have spoken to him or her*) x *is an extrovert*

¬ *someone who has spoken only to people who have spoken to him or her is such that (he or she is an extrovert)*

¬ *someone who has spoken only to people who have spoken to him or her is an extrovert*

No one who has spoken only to people who have spoken to him or her is an extrovert