8.2.2. Quantifier scope ambiguities

One of the points of section 7.1.1 was that a simple dichotomy is not sufficient to account for the possible ambiguities when more than two quantifier phrases are present. So we need to extend the ideas developed in 8.2.1 to distinguish more than a single pair of claims. Consider the example cited in 7.1.1:

Every reporter asked a question of each juror.

This could be said to generalize along two dimensions (reporters and jurors) and the exemplification (of a question that was asked) might be claimed to be uniform in either or both of these dimensions. That is, the exemplification may be set forth as doubly uniform (the same question could be used as an example in all cases) or as uniform in one dimension only (e.g., we might have to vary the question cited as an example from reporter to reporter but would not need to vary it from juror to juror provided we keep the reporter fixed). This means that there are four interpretations here: the basic claim of doubly general exemplification and three stronger claims, citing uniformity in or the other dimension or in both of them.

With three quantifier phrases, there will be six different symbolic representations of this sentence since there are three choices for the first quantifier phrase to be analyzed and, for each of these, two orders in which the remaining two can be analyzed. The results of these choices are shown below in a way that reflects their logical relations, with stronger claims lower on the page and equivalent claims grouped together.

(∀x: Rx) (∀y: Jy) (∃z: Qz) Axzy
(∀y: Jy) (∀x: Rx) (∃z: Qz) Axzy)
[no uniformity claimed]

(∀x: Rx) (∃z: Qz) (∀y: Jy) Axzy
[claims uniformity with respect to jurors]

(∀y: Jy) (∃z: Qz) (∀x: Rx) Axzy
[claims uniformity with respect to reporters]

(∃z: Qz) (∀x: Rx) (∀y: Jy) Axzy
(∃z: Qz) (∀y: Jy) (∀x: Rx) Axzy)
[claims uniformity with respect to both jurors and reporters]

[A: λxyz (x *asked* y *of* z); J: λx (x *is a juror*); Q: λx (x *is a question*); R: λx (x *is a reporter*)]

Two pairs of these six forms are equivalent, and the distinguishing features of the forms that are not equivalent is the location of the existential quantifier used to represent *a question*—whether it is outside the scope of one or the other of the universal quantifier phrases, outside the scope of both, or outside the scope of neither. For, when the two universals are side by side (and neither binds variables in the restriction of the other), we can interchange them without altering the proposition expressed. The four non-equivalent symbolic possibilities shown above correspond to the four possibilities of uniformity we have noticed.

This brings us to one of the chief lessons of this section. When a claim of general exemplification is uniform with respect to a given dimension of generality, the existential quantifier representing the claim of exemplification should have wider scope than the universal quantifier corresponding to the relevant dimension of generality. When you are faced with choosing the order in which to represent several quantifier phrases and you wonder what effect the order you choose will have on the meaning, you can proceed as follows. First, identify the quantifier phrases making existential claims and the quantifier phrases that generalize on one or another dimension. Then ask, for each existential quantifier phrase and each generalizing one, whether the existential claims that exemplification is uniform on the dimension referred to by the generalizing phrase. If the existential makes this claim, it should be dealt with first; if it does not, the generalizing quantifier phrase should be given wider scope. The answers to these questions will settle the relative order of treatment for each pair consisting of an existential and a universal.

This approach may not settle all questions about the order in which quantifier phrases are to be treated in claims of general exemplification, but the remaining questions can be settled arbitrarily without any effect on the meaning ascribed to the sentence. For example, if *a question* is held to claim an exemplification that is uniform with respect to both reporters and jurors, we know the existential quantifier phrase must be treated first. Nothing is implied about the order in which we go on to handle *every reporter* and *each juror*, but that order also has no affect on the content of the result.

The language we have been using to speak about the process of settling the relative scope of quantifier phrases is open to one sort of misinterpretation. Although there is no way to arrange overlapping scopes to claim uniformity in each of two dimensions without claiming uniformity in both, this does not mean that claims of uniformity in each of two dimensions together entail a claim of uniformity in both. Since there might be different examples exhibiting each sort of uniformity, there can be situations where both sorts of partial uniformity occur without full uniformity. For example, it may be that reporters had favorite questions and also that there was an obvious question for each juror while still there was no one question that appeared in all interviews. In short, while the conjunction

$$(\forall x\text{: } Rx)\ (\exists z\text{: } Qz)\ (\forall y\text{: } Jy)\ Axzy \land (\forall y\text{: } Jy)\ (\exists z\text{: } Qz)\ (\forall x\text{: } Rx)\ Axzy$$

says more than either of its conjuncts, it still says less than a claim of doubly uniform exemplification.

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