8. Numerations

8.1. The existential quantifier

8.1.0. Overview

A generalization quantifies only by claiming the absence of counterexamples. We will now turn claims that are more explicitly quantificational; the first are claims of the existence of examples, each of which asserts that there is at least one object of a certain sort.

8.1.1. Exemplification
Most of the ideas used in analyzing English generalizations apply also to claims of exemplification; but, instead of three forms, we have only one.

8.1.2. Obversion
As was noted in 7.3.1, every claim of existence amounts to the denial of a generalization.

8.1.3. Conversion
The quantifier phrase and quantified predicate of an existential claim are interchangeable, a feature that is associated with the use of the phrase *there is*.

8.1.4. Existentials exemplified
Most analyses of existential claims are straightforward, but there is often a great variety of ways of expressing the same content in English.

8.1.5. Existential commitment
The impact of the way undefined terms are being handled is clearest when we consider the content of existential claims.

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8.1.1. Exemplification

Although we have looked at quantification and quantifiers, the idea of quantity has not been much in evidence. Of course it could be found in discussions of generalization if we look hard enough because any generalization can be understood to claim that its counterexamples number 0. This way of looking at generalizations is rather forced, but the sorts of claims we will now consider can all be stated rather naturally by reference to numbers.

Our study will have a somewhat different character in another respect, too. We had to devote much effort to analyzing generalizations before we could put them into symbolic form, but once that analysis was carried out, the symbolic forms were easily stated. In this chapter, our symbolic analyses will require much less preparatory work on the English sentences. This is in part because we can carry over ideas from the last chapter, but it is in large part due to the relative simplicity of the means of expression we will encounter in English. However, before long, we will have considered quite a variety of numerical claims. Since most of these will be expressed using only one new symbol, we have to devote more of our attention to developing the symbolic means to represent English forms. Thus the focus of our attention will shift slightly, though noticeably, from English to the symbolic language.

The first evidence of this is that we will begin our discussion of our first new sort of logical form by considering its symbolic version. The ***unrestricted existential quantifier*** is an operator that applies to a one-place predicate abstract, its ***quantified predicate***, to say that the extension of the predicate contains at least one value, that it is ***non-empty***. We will use the sign ∃ (named ***there exists***) for this operation. A sentence $\exists[\lambda x\ \theta x]$ formed using this quantifier says that the predicate $\lambda x\ \theta x$ is ***exemplified***, that there is some value (in the range **R**) that serves as an example of a thing that $\lambda x\ \theta x$ is true of. Thus the sentence *Something fell* could be represented as $\exists[\lambda x\ Fx]$ (using F: λx (x *fell*)).

The ***restricted existential quantifier*** is used to claim the existence of examples that are not merely in the referential range but in some more specific class. It applies to a pair of one-place predicates, its ***restricting*** and ***quantified*** predicates, to form a sentence $\exists[\lambda x\ \rho x][\lambda x\ \theta x]$ that asserts that the extension of θ contains at least one member of the extension of ρ. So *Some dog climbs trees* could be represented as $\exists[\lambda x\ Dx][\lambda x\ Cx]$ (using D: λx (x *is a dog*); C: λx (x *climbs trees*)).

We will generally use abbreviations that, like those used for the universal quantifiers, suppress the symbol λ. Thus ∃ [λx θx] becomes ∃x θx, and ∃[λx ρx] [λx θx] becomes (∃x: ρx) θx. The two examples above could be written in this way as ∃x Fx and (∃x: Dx) Cx, respectively. We will continue to refer to the component formulas ρx and θx as the ***restricting*** and ***quantified*** formulas, respectively.

As with universals, we have principles of equivalence that enable us to restate restricted existentials as unrestricted existentials, and vice versa.

(∃x: ρx) θx ⇔ ∃x (ρx ∧ θx)
∃x θx ⇔ (∃x: x = x) θx

These should be compared to the analogous principles for the universal quantifiers discussed in 7.2.1. The only disanalogy appears in the first, which contains a conjunction at a point where the corresponding principle for universals contains a conditional.

The reason is this. While the restricting predicate serves with both universals and existentials to make the claim more specific or less general, this has a different effect on the strength of the claim—on how much is said—in the two cases. When a generalization is restricted, it generalizes about a more narrowly specified class, and its claim is weakened; it says less, and this is represented by the hedging effect of the conditional. On the other hand, when an existential claim is restricted, the kind of example claimed to exist is more fully specified and the claim is strengthened; it says more, and this is represented by the strengthening effect of conjunction.

In both of the English examples above, the quantifier phrases we analyzed had *some* as their quantifier word. This is not the only word that can signal the presence of an existential quantifier. In particular, as was discussed in 7.3.1, one of the chief uses of the indefinite article *a* is to claim the existence of an example, to make an ***existential claim*** or ***claim of exemplification***. Thus either *Some dog barked* or *A dog barked* could be used in English to express the existential claim represented symbolically by (∃x: Dx) Bx (using B: λx (x *barked*); D: λx (x *is a dog*)).

Although there is more than one way of expressing an existential claim, we do not have several kinds of existential claim in the way in which we have several kinds of generalization. That is, there is no quantifier word that indicates that the denial of the quantified predicate is being exemplified and none that indicates that the example is to be found outside the class picked out by the class indicator. At least, this is so if we follow the policy of 7.3.1 and analyze *not every* and *not only* rather than treating them as units. Of course, existential quantifiers can apply to negative predicates; but the corresponding English forms will be like our symbolic notation in having such negation as an explicit part of the quantified predicate or the class indicator instead of signaling the presence of negation by the quantifier word used.

There is one special problem concerning existential claims that deserves some discussion though it cannot be given a fully satisfactory treatment here. The word *some* is often used with plural noun phrases, as in *Some mice were in the attic*, and bare plural common noun phrases are sometime used to the same effect, as in *Mice were in the attic*. One would expect such sentences to claim the existence of multiple examples, but if we consider their implications rather than their implicatures, this does not seem to be so. Suppose you knew that one and only one mouse had been in the attic. If you were asked the question *Were mice in the attic?* the natural response would be *Yes, one was* rather than *No, only one was*. This suggests that we are prepared to count a sentence like *Mice were in the attic* as true even when there is only one example—although it would generally be misleading to assert it under such conditions.

There is another argument for the same conclusion. Under one interpretation of it, the ambiguous sentence *Mice were not in the attic* is the denial of *Mice were in the attic*. And, so understood, it is equivalent to *No mice were in the attic*. But *No mice were in the attic* and *No mouse was in the attic* are both negative generalizations that make the same claim: that there is no example to be found among mice of a thing that was in the attic. The moral is that the distinction between singular and plural in English escapes our analysis. This is not to say that we have no way to represent claims that actually *imply* the existence of multiple examples; we will encounter quite a variety beginning in 8.3.2.

8.1.2. Obversion

Just as generalizations deny the existence of counterexamples, denials of generalizations claim the existence of such examples. This suggests that it should be possible to restate the denials of generalizations as existential claims. And it is not hard to see how. For example, *Not every dog barks* claims the existence of an example among dogs of something that does not bark, so it is equivalent to *Some dog does not bark* (or *Some dogs do not bark*). And *Not only trucks were advertised* claims the existence of a non-truck that was advertised, so it is equivalent to *Some non-truck was advertised*. The general principle behind these equivalences takes the form

$$\neg (\forall x\!: \rho x)\ \theta x \Leftrightarrow (\exists x\!: \rho x)\ \overline{\theta x}$$

To deny that the predicate θ is true generally of the extension of ρ (which is what $\neg (\forall x\!: \rho x)\ \theta x$ does) is to claim the existence, in the extension of ρ, of a counterexample—i.e., an object of which the predicate $\lambda x\ \overline{\theta x}$ is true. And this is just what $(\exists x\!: \rho x)\ \overline{\theta x}$ claims. This is one form of a principle for which we will adapt the traditional term ***obversion***. (This term is usually applied more narrowly to equivalences where the generalization is direct and where the negation is part of a noun phrase in one of the two equivalent sentences—each of our examples fails on one of these scores.) The bar notation functions here as before to provide a single notation for both adding and removing negation. With it, the principle says that the denial of a negative generalization is equivalent to a claim of exemplification for either a doubly negative or an affirmative predicate. The sentence *Not everyone failed to laugh* is equivalent to *Someone laughed* as well as to *Someone did not fail to laugh*.

A second form of obversion can be found in the possibility of using a generalization to deny an existential claim. To deny *Some dog climbs trees*, we can assert *No dog climbs trees*. And, in general, to deny the existence of an example, we can make an appropriate negative generalization:

$$\neg (\exists x\!: \rho x)\ \theta x \Leftrightarrow (\forall x\!: \rho x)\ \overline{\theta x}$$

The two forms of obversion for restricted quantifiers are matched by two forms for unrestricted quantifiers and we can use some notation introduced in 7.3.2 to state the principles for both sorts of quantifiers at once:

$$\neg (\forall x\ldots)\ \theta x \Leftrightarrow (\exists x\ldots)\ \overline{\theta x}$$
$$\neg (\exists x\ldots)\ \theta x \Leftrightarrow (\forall x\ldots)\ \overline{\theta x}$$

That is, to deny that a predicate is universal is to say that its negation is exemplified; and to deny that a predicate is exemplified is to say that its negation is universal.

The second form of obversion shows the equivalence of the two sorts of analysis that we can now give for many uses of *any* (when it contrasts with *every*). The following repeats and extends an example of 7.3.3 :

Tom didn't see anything
Everything is such that (Tom didn't see it)
∀x (*Tom didn't see* x)
∀x ¬ *Tom saw* x

∀x ¬ Stx

Tom didn't see anything
¬ *Tom saw something*
¬ *something is such that (Tom saw it)*
¬ ∃x *Tom saw* x

¬ ∃x Stx

[S: λxy (x *saw* y); t: *Tom*]

These two symbolic forms are often equally close to the forms of English sentences. Although negated existentials preferable to negative generalizations for the purposes of the exercises in this chapter, the role of negative generalizations in deductive reasoning is clearer both intuitively and in the context of the system of derivations we will use.

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8.1.3. Conversion

The restricted existential $(\exists x\colon \rho x)\ \theta x$ asserts that the extension of θ contains at least one member of the extension of ρ. This is to say that the two extensions overlap, that their intersection is non-empty. The overlapping of extensions is a symmetric relation; and, as this suggests, $(\exists x\colon \rho x)\ \theta x$ and $(\exists x\colon \theta x)\ \rho x$ are equivalent. This principle asserting this,

$$(\exists x\colon \rho x)\ \theta x \Leftrightarrow (\exists x\colon \theta x)\ \rho x$$

is known traditionally as ***conversion***. Its truth can be confirmed by recalling that the two sentences it relates are equivalent to the unrestricted forms $\exists x\ (\rho x \wedge \theta x)$ and $\exists x\ (\theta x \wedge \rho x)$ and that the latter two are equivalent by the principle commutativity for conjunction.

Conversion indicates that the restricting and quantified predicates have a symmetric role in an existential claim. Since the function of the restricting predicate is served in English by a common noun phrase, to exhibit conversion in English we must move between a common noun phrase and a predicate, perhaps converting the common noun phrase to a predicate using the phrase *is a*, or converting the predicate to a common noun phrase using a device such as *thing that*. Thus *Some dog climbs trees* can be rephrased as *Some**thing that** climbs trees **is a** dog*. More natural examples of conversion are to be found in sentences that assert the overlapping of two classes: *Some mammal is an aquatic animal* is equivalent to *Some aquatic animal is a mammal*.

The symmetry between restricting and quantified predicates in existential claims suggests that we could consider an unrestricted existential equally well as an existential without a restricting predicate or as one with a restricting predicate but without a quantified predicate. Indeed, the latter provides a fair description of one sort of English existential. Sentences like *There is a problem* have a peculiar grammar that confounds the ways we have so far dealt with quantificational claims, for there is no natural way of analyzing it into a quantifier phrase and a quantified predicate. It could be held to contain the quantifier phrase *a problem*, but λx (*There is* x) is not a genuine predicate and rephrasing it as λx (x *is there*) is of little help. If we try to state its symbolic analysis directly, it clearly should be something like ∃x (x *is a problem*), for it says that the predicate λx (x *is a problem*) is exemplified. If we put this symbolic form back into English, we get *Something is a problem*. And, in general, existential claims of the form *there is a C* can be treated symbolically by restating *there* as *something* (or perhaps *someone* or the like when a contextual bound on the intended sort of example is made explicit). More precisely, we take the class indicator of the *there-is* existential, add the phrase *is a* to make it into a predicate, and supply *something* (or *someone*) as the subject.

We can go a little way below the surface of the rule of thumb just stated (though we will still be naïve from a grammarian's point of view). If we are to find a quantified predicate in a sentence like *There is a problem*, it must be one that contributes nothing to the claim being made. That means it must be a predicate like $\lambda x\ (x = x)$ or $\lambda x\ \top$ that is universal as a matter of logic. Compare *There is a problem* to a sentence like *There ensued an argument*. Grammarian's tend to view the latter as a variant on *An argument ensued* so we might connect the former in a similar way to *A problem is*. And if we can make sense of λx (x *is*) at all, we might end up regarding it as a universal predicate (though the discussion of existential commitment at the end of this section will suggest that there is room for controversy here). This approach would lead us to something like

(∃x: x *is a problem*) ⊤

as a first step in our analysis of the *there-is* existential. Applying conversion would then get us (∃x: ⊤) x *is a problem*, which can be restated as ∃x x *is a problem* if we use an unrestricted existential quantifier.

In this sort of example, we have taken a roundabout way to the result we reached by the expedient of restating *there* as *something*. There are other cases, however, where the more complex approach is more helpful. For example, we would not want to simply replace *there* by *something* in *There are three things that you need to remember*, but rephrasing the latter as *Three things that you need to remember are*, however odd as English, would point us in the direction of the correct analysis. (In section 8.3.2, we will discuss the analysis of phrases, such as *three things that you need to remember*, that have the form *n Cs*.)

However peculiar they are in their logical grammar, *there-is* existentials are not oddities. They are quite common, in part because they can help us to avoid the sort of ambiguities of quantifier scope noted in 7.1.1 (and to be discussed again in 8.2.1).

8.1.4. Existentials exemplified

The following examples introduce no new problems. They simply illustrate the use of the existential quantifier in analyzing two pairs of equivalent sentences.

Ann sent a package and Bill received it
A package is such that (Ann sent it and Bill received it)
(∃x: x *is a package*) (*Ann sent* x *and Bill received* x)
(∃x: Px) (*Ann sent* x ∧ *Bill received* x)

$$(\exists x\colon Px)\ (Sax \wedge Rbx)$$
$$\exists x\ (Px \wedge (Sax \wedge Rbx))$$

Bill received a package that Ann sent
A package that Ann sent is such that (Bill received it)
(∃x: x *is a package that Ann sent*) *Bill received* x
(∃x: x *is a package* ∧ *Ann sent* x) Rbx

$$(\exists x\colon Px \wedge Sax)\ Rbx$$
$$\exists x\ ((Px \wedge Sax) \wedge Rbx)$$

[P: λx (x *is a package*); S: λxy (x *sent* y); R: λxy (x *received* y); a: *Ann*; b: *Bill*]

Some people have not seen Crawfordsville
Some people are such that (they have not seen Crawfordsville)
(∃x: x *is a person*) x *has not seen Crawfordsville*
(∃x: Px) ¬ x *has seen Crawfordsville*

$$(\exists x\colon Px)\ \neg Sxc$$
$$\exists x\ (Px \wedge \neg Sxc)$$

There are people who have not seen Crawfordsville
Something is a person who has not seen Crawfordsville
∃x x *is a person who has not seen Crawfordsville*
∃x (x *is a person* ∧ x *has not seen Crawfordsville*)
∃x (x *is a person* ∧ ¬ x *has seen Crawfordsville*)

$$\exists x\ (Px \wedge \neg Sxc)$$

[W: λx (x *is a person*); A: λxy (x *has seen* y); c: *Crawfordsville*]

Notice that the transition from the first to the second and from the third to the fourth is made by absorbing a part or all of the quantified predicate as modifier in the class indicator. The second transition is licensed by the equivalence that governs restatement using unrestricted quantifiers, and the first transition is licensed by the following principle (read from right to left)

$$(\exists x\colon \rho x)\ (\pi x \wedge \theta x) \Leftrightarrow (\exists x\colon \rho x \wedge \pi x)\ \theta x$$

When read right to left, this amounts to an extended form of the principle governing the restatement of restricted quantifiers since it tells us that a conjunct of the restricting formula may be instead conjoined with the quantified formula. If we were to take the absorption of content into the quantifier phrase one step further we would arrive at the form $\exists x\ (\rho x \wedge (\pi x \wedge \theta x))$ or, in this case, at the sentence *There is a package that Ann sent and Bill received*.

The equivalence displayed above also explains why the distinction between restrictive and non-restrictive relative clause, which can be very important for generalizations, disappears in the case of existential quantifier phrases. Contrast the difference between the generalizations *Mammals that are aquatic are large* and *Mammals, which are aquatic, are large* with the equivalence of the existential claims *A man who is carrying a box is at the door* and *A man, who is carrying a box, is at the door*. The latter could be given the following analyses.

A man who is carrying a box is at the door
A man who is carrying a box is such that (he is at the door)
(∃x: x *is a man who is carrying a box*) x *is at the door*
(∃x: x *is a man* ∧ x *is carrying a box*) x *is at the door*
(∃x: Mx ∧ *a box is such that (*x *is carrying it)*) Axd
(∃x: Mx ∧ (∃y: y *is a box*) x *is carrying* y) Axd

$$(\exists x\colon Mx \wedge (\exists y\colon By)\ Cxy)\ Axd$$

A man, who is carrying a box, is at the door
A man is such that (he is carrying a box and he is at the door)
(∃x: x *is a man*) x *is carrying a box and* x *is at the door*
(∃x: Mx) (x *is carrying a box* ∧ x *is at the door*)
(∃x: Mx) (*a box is such that (*x *is carrying it)* ∧ Axd)
(∃x: Mx) ((∃y: y *is a box*) x *is carrying* y ∧ Axd)

$$(\exists x\colon Mx)\ ((\exists y\colon By)\ Cxy \wedge Axd)$$

[A: λxy (x *is at* y); C: λxy (x *is carrying* y); M: λx (x *is a man*); d: *the door*]

Moving all the information about the example claimed to exist to the quantified formula would leave us with

$$\exists x\, (Mx \wedge (\exists y\!: By)\, Cxy \wedge Axd)$$

if we ignore the grouping of the conjuncts. This might be expressed in English as *There is a man, who is carrying a box and is at the door*. The corresponding sentence with a restrictive relative clause, *There is a man who is carrying a box and is at the door*, would say the same thing, but it would be more naturally expressed by stating the various properties of the example in a different order—e.g., *There is a man at the door who is carrying a box*.

If we restate the second existential using an unrestricted quantifier, we obtain

$$\exists x\, (Mx \wedge \exists y\, (By \wedge Cxy) \wedge Axd)$$

which is the form of the (slightly awkward) English sentence *There is a man and there is a box he is carrying and he is at the door*, which cannot be analyzed as a conjunction because of the pronouns *he*. While we cannot give the main existential narrower scope than conjunction, it is possible to give the second existential wider scope and write (after some regrouping of conjuncts)

$$\exists x\, \exists y\, ((Mx \wedge By) \wedge (Cxy \wedge Axd))$$

This can be thought of as the analysis of *There is a man and a box and the man is carrying the box and is at the door*, where *the man* and *the box* serve to mark cross reference, or of the analogous sentence *There is a man and a box and he is carrying it and is at the door*, where we use ordinary pronouns instead.

This last form claims the existence of a pair of objects exemplifying the relation λxy (x *is a man and* y *is a box and* x *is carrying* y *and is at the door*). That comes to the same thing as claiming the existence of a man and box which exemplify the relation λxy (x *is carrying* y *and is at the door*), something that can be expressed symbolically by using a pair of restricted quantifiers:

$$(\exists x\!: Mx)\, (\exists y\!: By)\, (Cxy \wedge Axd)$$

This may have no very natural English rendering but it can be expressed by *Some man and box are such that he is carrying it and is at the door*.

The form of restatement used in the last two cases—that is, expanding the scope of an existential to include the whole of a conjunction when it will bind no variables in the other conjuncts—is always possible. And, of course, the opposite operation—narrowing the scope of an existential to the conjuncts of a conjunction in which it actually binds variables—is equally legitimate. Looked at from the latter point of view, the following equivalences (where ϕ has no free occurrence of x and (∃x...) is either a restricted or unrestricted quantifier)

$$(\exists x...)\, (\phi \wedge \theta x) \Leftrightarrow \phi \wedge (\exists x...)\, \theta x$$
$$(\exists x...)\, (\theta x \wedge \phi) \Leftrightarrow (\exists x...)\, \theta x \wedge \phi$$

can be described as ***confinement principles***, as can obversion.

The change between universal to existential along with confinement in obversion is the exception rather than the rule; confinement of unrestricted quantifiers (both existential and universal) is possible in most other cases following the lines shown above. (On the other hand, it is not always possible to confine the scope of restricted quantifiers to components in which they bind variables. For example, (∀x: ρx) (ϕ ∧ θx) is not equivalent to ϕ ∧ (∀x: ρx) θx; the first is true and the second is false in a case where ϕ is false but the extension of ρ is empty, for then there can be no counterexample to a generalization over than extension.)

Apart from negations, the only locus of confinement that forces a change between universals and existentials is the antecedent of a conditional. That is also a location where *any* can be used in contrast with *every*, and one of the forms of confinement declares the equivalence of the two natural analyses of such a sentence. Here are the two approaches in the case of an example from 7.3.3

If anyone backs out, the trip will be canceled
Everyone is such that (if he or she backs out, the trip will be canceled)
(∀x: x *is a person*) (*if* x *backs out, the trip will be canceled*)
(∀x: Px) (x *will back out* → *the trip will be canceled*)

$$(\forall x\!: Px)\, (Bx \rightarrow Ct)$$
$$\forall x\, (Px \rightarrow (Bx \rightarrow Ct))$$

If anyone backs out, the trip will be canceled
Someone will back out → *the trip will be canceled*
Someone is such that (he or she will back out) → *the trip will be canceled*
(∃x: x *is a person*) x *will back out* → Ct

$$(\exists x\!: Px)\, Bx \rightarrow Ct$$
$$\exists x\, (Px \wedge Bx) \rightarrow Ct$$

[B: λx (x *will back out*); C: λx (x *will be canceled*); P: λx (x *is a person*); t: *the trip*]

This examples illustrates the following general confinement principle (where, as before, ϕ must contain no free occurrences of x):

$$(\forall x\ldots)\ (\theta x \rightarrow \phi) \Leftrightarrow (\exists x\ldots)\ \theta x \rightarrow \phi$$

Note that this principle concerns only cases where variables bound by the quantifier are limited to the antecedent of a conditional; confinement to the consequent of a conditional follows the same pattern as confinement to a component of a conjunction or disjunction. The principle is also limited to cases where the quantifiers with wide scope are universal; an unrestricted existential with wide scope can be confined to the antecedent of a conditional (provided it is changed to a universal)

$$\exists x\ (\theta x \rightarrow \phi) \Leftrightarrow \forall x\ \theta x \rightarrow \phi$$

but a restricted existential cannot be confined in this way without weakening the claim being made.

The analogies between restricted universals and conditionals and the possibility of a contrast between *any* and *every* when a quantifier phrase appears within the quantifier phrase of a generalization should suggest that a confinement principle might hold also in such a case. A principle of this sort is illustrated by the following two equivalent analyses of an example from 7.4.2:

Everything that is relevant to anything is worth knowing
Everything is such that (everything that is relevant to it is worth knowing)
∀x *everything that is relevant to* x *is worth knowing*
∀x *everything that is relevant to* x *is such that (it is worth knowing)*
∀x (∀y: y *is relevant to* x) y *is worth knowing*

$$\forall x\ (\forall y\colon Ryx)\ Wy$$
$$\forall x\ \forall y\ (Ryx \rightarrow Wy)$$

Everything that is relevant to anything is worth knowing
Everything that is relevant to something is such that (it is worth knowing)
(∀y: y *is relevant to something*) y *is worth knowing*
(∀y: *something is such that (*y *is relevant to it)*) y *is worth knowing*
(∀y: ∃x y *is relevant to it* x) Wy

$$(\forall y\colon \exists x\ Ryx)\ Wy$$
$$\forall y\ (\exists x\ Ryx \rightarrow Wy)$$

[R: λxy (x *is relevant to* y); W: λx (x *is worth knowing*)]

In this case, the general confinement principle takes the form

$$(\forall x\ldots)\ (\forall y\colon \rho xy)\ \theta y \Leftrightarrow (\forall y\colon (\exists x\ldots)\ \rho xy)\ \theta y$$

where the formula ρxy may contain free occurrences of the variable x as well as y but θy may not contain free occurrences of x (and any restriction on the quantifiers (∀x...) and (∃x...) may not contain free occurrences of y).

Similarly, confinement of an existential within the restricting formula of an existential is possible when all the variables it binds are in that formula (and requires no change to a universal). Indeed, we might regard *A man who is carrying a box is at the door* as the result of applying such a principle to *A box is such that some man who is carrying it is at the door*.

On the other hand, there is no analogous principle for an existential that binds variables only in the restriction of a universal because confining such a quantifier would involve reversing the relative scope of an existential and a universal and could alter meaning in ways to be discussed in 8.2. And there is also no general principle for confining universals to existentials under similar conditions.

8.1.5. Existential commitment

To non-logicians this heading may suggest a certain sort of moral (or quasi-moral) seriousness; but, to a logician, the phrase means roughly ‘implication of exemplification’. That is, there is an existential commitment when there is an implication that a predicate is exemplified or that a certain thing or kind of thing exists.

A *there-is* sentence is probably the most explicit way of taking on an existential commitment in the logician’s sense. And it might be doubted that we have shown proper respect to this sort of sentence and to other existentials. The problem can be sharpened by thinking about the name *Santa Claus*. The analysis of the sentence *There is a Santa Claus* raises issues that would be distracting at this point, but enough has been said already to suggest that we might analyze *There is something that is Santa Claus* as ∃x x = s (with s abbreviating *Santa Claus*). But is this analysis right? The sentence ∃x x = s is a tautology, for it says that there is some referential value that is identical to the value of s, and that is bound to be true, even if s is undefined. So on this analysis, we end up saying that the sentence *There is something that is Santa Claus* is indubitably true (but we also say it is empty of content, so we have no genuine reassurance to offer small children).

This empty existential commitment is not as crazy as it may seem. We have interpreted the existential quantifier as claiming the existence of examples among referential values, and the nil value—the reference value of non-referring terms—is a genuine referential value. Since this interpretation of the existential quantifier is just a stipulation of the meaning of the sign ∃, there is really no way to quarrel with it. But things may heat up when we use this special sign to render the English *there-is* form and other existential sentences. That is, it can still be asked whether English existentials claim merely that examples may be found among reference values or make the stronger claim that examples can be found among non-nil values. Let us refer to the latter, more specific sort of claim as a ***substantive existential commitment***.

Looking at bare *there-is* existentials may sharpen the issue in the wrong way so let us look at other cases. We can attribute a substantive existential commitment to a form (∃x: ρx) θx if ρ is necessarily false of the nil value; for then any example in the extension of ρ must then be a non-nil value. And the same is true of the form ∃x θx if the extension of θ is necessarily limited to objects. The difficulty with ∃x (x = s) is that there seems to be nothing to force a similar limitation since we have already stipulated the extension of =; it is the only predicate in this sentence, and we have stipulated that it holds of the nil value and itself. However, we may have placed too simple an interpretation on the question of whether there is a Santa Claus; perhaps a child is really asking whether there was some *person* who is Santa Claus. We can analyze the sentence *There is someone who is Santa Claus* as ∃x (Px ∧ x = s) [P: λx (x *is a person*); s: *Santa Claus*], and this is not a tautology. The substantive existential commitment here is imposed by the predicate P.

These are controversial matters; and, although the approach we have taken to *there-is* existentials is a viable one, it is not the only viable one. Accordingly, it is worth noting that we have the resources available to take a different approach. If we wish to attribute substantive existential commitment through purely logical vocabulary, we could introduce a logical constant to capture the predicate λx (x *is non-nil*), and we would stipulate that the extension of such a constant on any range **R** consist of all non-nil values. One alternative to the analyses of claims of exemplification that we have been giving is then that “real” claims of exemplification (and “real” generalizations) always have such predicate as part of their restrictions. Another way of formulating this alternative approach would be to introduce an individual term that is stipulated to refer to the nil value—i.e., one whose reference is stipulated to be undefined. Substantial existential commitment could then be expressed by denying identity with this term. (In fact, such a term will be a by-product of the approach to definite descriptions we consider in 8.4.2, but we will not make it part of our analysis of claims of exemplification.)

8.1.s. Summary

Generalizations do not make claims about quantity in any very explicit way, and we are now considering sentences that do. We will refer to the claims they make as existential claims or claims of exemplification. The unrestricted existential quantifier says that the predicate it applies to is exemplified—i.e., it has a non-empty extension. The restricted existential quantifier says that its quantified predicate is exemplified within the extension of its restricting predicate—i.e., the intersection of their extensions is non-empty. Both use the sign ∃ (there exists) and we will refer to sentences formed with either as existentials. An unrestricted existential can be restated as a restricted existential whose restricting predicate is universal, and a restricted existential can be restated by applying an unrestricted existential to a predicate formed from the restricting and quantified predicates using conjunction (note: *not* using the conditional). Although English existentials can appear with either singular or plural quantifier phrases, this does not seem to affect the proposition expressed and the difference will not be captured in our analyses.

To deny a generalization is to claim the existence of a counterexample, and this suggests that the negation of a universal should be equivalent to an existential with a negative quantified predicate. This is so, and the negation of an existential is also equivalent to a negative generalization. We extend the traditional term obversion to both principles.

Another traditional principle is conversion, which tells us that we can interchange the restricting and quantified predicates of a restricted existential. This suggests that we could regard the single predicate in an unrestricted existential as either a restricting or a quantified predicate. That provides some explanation of English *there-is* existentials, which can have class indicators without quantified predicates. A rule of thumb for handling the simpler examples of such sentences is to replace *there* by *something* (or *someone*).

English sentences that claim the existence of the same sort of example can vary widely in the way the properties this example is said to have are distributed between the quantifier phrase and quantified predicate. The logical equivalence of different ways of distributing this information explains why the difference between restrictive and non-restrictive relative clauses does not matter when they modify the class indicator of an existential quantifier phrase. Other forms of equivalent restatement are the result of confining the scope of an existential to a formula in which all its bound variables appear. Confinement principles sometimes require a change between universal and existential quantifiers, and this explains why *any* can sometimes be treated either by a universal with wide scope or an existential with narrow scope.

Any existential sentence—indeed any sentences that entail an existential—can be said to involve an existential commitment, but the examples whose existence make existentials true can be any referential values, even the nil value. This may seem to conflict with the substantive existential commitment, to objects rather than mere referential values, that many find in English existentials. This commitment might be traced to the logical properties of non-logical vocabulary; but, if that account is rejected, it is possible to introduce a logical predicate that carries the commitment (through a stipulation that its extension includes only non-nil values).

Glen Helman 21 Nov 2004

8.1.x. Exercise questions

1. Analyze the sentences below in as much detail as possible. For the most practice using existentials, avoid using universals in your analyses.

- **a.** *Someone is missing*.
- **b.** *No one found the loot*.
- **c.** *There is a tavern in the town*.
- **d.** *Some winner of the lottery has not come forward*.
- **e.** *Tod watched a dance troop from India*.
- **f.** *The search turned up no car fitting the description*.
- **g.** *There is a button behind you that will open the door*.
- **h.** *If Tom doesn't find anything, he'll be disappointed*.
- **i.** *Al went to a restaurant no one he knew had heard of*.

2. Synthesize idiomatic English sentences that express the propositions associated with the logical forms below by the intensional interpretations that follow them.

a.	∃x Bx	[B: λx (x *is burning*)]
b.	(∃x: Px) Axd	[A: λxy (x *is at* y); P: λx (x *is a person*); d: *the door*]
c.	(∃x: Fx) Rtx	[F: λx (x *is a fire*); R: λxy (x *reported* y); t: *Tamara*]
d.	¬ (∃x: Px ∧ Nxr) Kxs	[K: λxy (x *knew* y); N: λxy (x *was in* y); r: *the room*; s: *Sam*]
e.	(∃x: Vx) (Tvx ∧ Sx)	[S: λx (x *shattered*); T: λxy (x *touched* y); V: λx (x *is a vase*); v: *Vic*]
f.	∃x (Hx ∧ Ljx)	[H: λx (x *had happened*); L: λxy (x *left to deal with* y); j: *Jane*]
g.	∃x (Fax ∧ Rbx)	[F: λxy (x *forgot* y); R: λxy (x *remembered* y); a: *Ann*; b: *Bill*]
h.	(∃x: Fx ∧ Hx) Dix	[D: λxy (x *detected* y); F: λx (x *was fast*); H: λx (x *was heavy*); i: *the instrument*]

8.1.xa. Exercise answers

1. **a.** *Someone is missing*
(∃x: x *is a person*) x *is missing*

(∃x: Px) Mx
∃x (Px ∧ Mx)

[M: λx (x *is missing*); P: λx (x *is a person*)]

b. *No one found the loot*.
¬ *someone found the loot*
¬ *someone is such that (he or she found the loot)*
¬ (∃x: x *is a person*) x *found the loot*

¬ (∃x: Px) Fxl
¬ ∃x (Px ∧ Fxl)

[F: λxy (x *found* y); P: λx (x *is a person*); l: *the loot*]

c. *There is a tavern in the town*
Something is a tavern in the town
Something is such that (it is a tavern in the town)
∃x x *is a tavern in the town*
∃x (x *is a tavern* ∧ x *is in the town*)

∃x (Tx ∧ Ixt)

[I: λxy (x *is in* y); T: λx (x *is a tavern*); t: *the town*]

It would also be possible to understand *in the town* to modify the verb *is* rather the noun *tavern*. In that case, the sentence could be restated as *A tavern is in the town* and be analyzed using a restricted existential.

d. *Some winner of the lottery has not come forward*
Some winner of the lottery is such that (he or she has not come forward)
(∃x: x *is a winner of the lottery*) x *has not come forward*
(∃x: x *is a winner of the lottery*) ¬ x *has come forward*

(∃x: Wxl) ¬ Fx
∃x (Wxl ∧ ¬ Fx)

[F: λx (x *has come forward*); W: λxy (x *is a winner of* y); l: *the lottery*]

e. *Tod watched a dance troop from India*
A dance troop from India is such that (Tod watched it)
(∃x: x *is a dance troop from India*) Tod *watched* x
(∃x: x *is a dance troop* ∧ x *is from* India) Wtx

(∃x: Dx ∧ Fxi) Wtx
∃x ((Dx ∧ Fxi) ∧ Wtx)

[D: λx (x *is a dance troop*); F: λxy (x *is from* y); W: λxy (x *watched* y); i: *India*; t: *Tod*]

f. *The search turned up no car fitting the description*
¬ *the search turned up a car fitting the description*
¬ *a car fitting the description is such that (the search turned it up)*
¬ (∃x: x *is a car fitting the description*) the search *turned up* x
¬ (∃x: x *is a car* ∧ x *fit* the description) Tsx

¬ (∃x: Cx ∧ Fxd) Tsx
¬ ∃x ((Cx ∧ Fxd) ∧ Tsx)

[C: λx (x *is a car*); F: λxy (x *fit* y); T: λxy (x *turned up* y); d: *the description*; s: *the search*]

g. *There is a button behind you that will open the door*
Something is a button behind you that will open the door
Something is such that (it is a button behind you that will open the door)
∃x x *is a button behind you that will open the door*
∃x (x *is a button behind you* ∧ x *will open* the door)
∃x ((x *is a button* ∧ x *is behind* you) ∧ Oxd)

∃x ((Bx ∧ Hxo) ∧ Oxd)

[B: λx (x *is a button*); H: λxy (x *is behind* y); O: λxy (x *will open* y); d: *the door*; o: *you*]

If the prepositional phrase *behind you* is understood to modify *is* instead of *button*, the sentence could be restated as *A button that will open the door is behind you*. This sentence would be analyzed by the restricted existential (∃x: Bx ∧ Oxd) Hxo, in which two of the conjuncts from the quantified predicate in the analysis above appear instead in the restriction of the quantifier.

h. *If Tom doesn't find anything, he'll be disappointed*
Tom won't find anything → *Tom will be disappointed*
¬ *Tom will find something* → Tom *will be disappointed*
¬ *something is such that (Tom will find it)* → Dt
¬ ∃x Tom *will find* x → Dt

¬ ∃x Ftx → Dt

[D: λx (x *will be disappointed*); F: λxy (x *will find* y); t: *Tom*]

i. *Al went to a restaurant no one he knew had heard of*
A restaurant no one Al knew had heard of is such that (Al went to it)
(∃x: x *is a restaurant no one Al knew had heard of*) Al *went to* x
(∃x: x *is a restaurant* ∧ *no one Al knew had heard of* x) Wax
(∃x: Rx ∧ ¬ *someone Al knew had heard of* x) Wax
(∃x: Rx ∧ ¬ *someone Al knew is such that (he or she had heard of* x*)*) Wax
(∃x: Rx ∧ ¬ (∃y: y *is a person Al knew*) y *had heard of* x) Wax
(∃x: Rx ∧ ¬ (∃y: y *is a person* ∧ Al *knew* y) Hyx) Wax

(∃x: Rx ∧ ¬ (∃y: Py ∧ Kay) Hyx) Wax
∃x ((Rx ∧ ¬ ∃y ((Py ∧ Kay) ∧ Hyx)) ∧ Wax)

H: λxy (x *had heard of* y); K: λxy (x *knew* y); P: λx (x *is a person*); R: λx (x is a restaurant); W: λxy (x *went to* y); a: *Al*]

2. **a.** ∃x x *is burning*
something is such that (it is burning)

Something is burning
or: There is something burning

b. (∃x: x *is a person*) x *is at the door*
someone is such that (he or she is at the door)

Someone is at the door

c. (∃x: x *is a fire*) *Tamara reported* x
Some fire is such that (Tamara reported it)

Tamara reported a fire

d. ¬ (∃x: x *is a person* ∧ x *was in the room*) x *knew Sam*
¬ (∃x: x *was a person in the room*) x *knew Sam*
¬ *someone in the room is such that (he or she knew Sam)*
¬ *someone in the room knew Sam*

No one in the room knew Sam

e. (∃x: x *is a vase*) (*Vic touched* x ∧ x *shattered*)
(∃x: x *is a vase*) (*Vic touched* x *and* x *shattered*)
A vase is such that (Vic touched it and it shattered)

Vic touched a vase and it shattered

f. ∃x (x *had happened* ∧ *Jane left to deal with* x)
∃x x *had happened and Jane left to deal with* x
something is such that (it had happened and Jane left to deal with it)

Something had happened and Jane left to deal with it

g. ∃x (*Ann forgot* x ∧ *Bill remembered* x)
∃x (*Ann forgot* x *and Bill remembered* x)
something is such that (Ann forgot it and Bill remembered it)

Ann forgot something and Bill remembered it
or: *There is something that Ann forgot and Bill remembered*

h. (∃x: x *was fast* ∧ x *was heavy*) *the instrument detected* x
(∃x: x *was fast and heavy*) *the instrument detected* x
(∃x: x *is a thing that was fast and heavy*) *the instrument detected* x
Something that was fast and heavy was such that (the instrument detected it)

The instrument detected something that was fast and heavy
or: *The instrument detected something fast and heavy*