8.1.4. Existentials exemplified

The following examples introduce no new problems. They simply illustrate the use of the existential quantifier in analyzing two pairs of equivalent sentences.

Ann sent a package and Bill received it
A package is such that (Ann sent it and Bill received it)
(∃x: x *is a package*) (*Ann sent* x *and Bill received* x)
(∃x: Px) (*Ann sent* x ∧ *Bill received* x)

(∃x: Px) (Sax ∧ Rbx)
∃x (Px ∧ (Sax ∧ Rbx))

Bill received a package that Ann sent
A package that Ann sent is such that (Bill received it)
(∃x: x *is a package that Ann sent*) *Bill received* x
(∃x: x *is a package* ∧ *Ann sent* x) Rbx

(∃x: Px ∧ Sax) Rbx
∃x ((Px ∧ Sax) ∧ Rbx)

[P: λx (x *is a package*); S: λxy (x *sent* y); R: λxy (x *received* y); a: *Ann*; b: *Bill*]

Some people have not seen Crawfordsville
Some people are such that (they have not seen Crawfordsville)
(∃x: x *is a person*) x *has not seen Crawfordsville*
(∃x: Px) ¬ x *has seen Crawfordsville*

(∃x: Px) ¬ Sxc
∃x (Px ∧ ¬ Sxc)

There are people who have not seen Crawfordsville
Something is a person who has not seen Crawfordsville
∃x x *is a person who has not seen Crawfordsville*
∃x (x *is a person* ∧ x *has not seen Crawfordsville*)
∃x (x *is a person* ∧ ¬ x *has seen Crawfordsville*)

∃x (Px ∧ ¬ Sxc)

[W: λx (x *is a person*); A: λxy (x *has seen* y); c: *Crawfordsville*]

Notice that the transition from the first to the second and from the third to the fourth is made by absorbing a part or all of the quantified predicate as modifier in the class indicator. The second transition is licensed by the equivalence that governs restatement using unrestricted quantifiers, and the first transition is licensed by the following principle (read from right to left)

$$(\exists x{:}\ \rho x)\ (\pi x \wedge \theta x) \Leftrightarrow (\exists x{:}\ \rho x \wedge \pi x)\ \theta x$$

When read right to left, this amounts to an extended form of the principle governing the restatement of restricted quantifiers since it tells us that a conjunct of the restricting formula may be instead conjoined with the quantified formula. If we were to take the absorption of content into the quantifier phrase one step further we would arrive at the form $\exists x\ (\rho x \wedge (\pi x \wedge \theta x))$ or, in this case, at the sentence *There is a package that Ann sent and Bill received*.

The equivalence displayed above also explains why the distinction between restrictive and non-restrictive relative clause, which can be very important for generalizations, disappears in the case of existential quantifier phrases. Contrast the difference between the generalizations *Mammals that are aquatic are large* and *Mammals, which are aquatic, are large* with the equivalence of the existential claims *A man who is carrying a box is at the door* and *A man, who is carrying a box, is at the door*. The latter could be given the following analyses.

A man who is carrying a box is at the door
A man who is carrying a box is such that (he is at the door)
(∃x: x *is a man who is carrying a box*) x *is at the door*
(∃x: x *is a man* ∧ x *is carrying a box*) x *is at the door*
(∃x: Mx ∧ *a box is such that (*x *is carrying it)*) Axd
(∃x: Mx ∧ (∃y: y *is a box*) x *is carrying* y) Axd

$$(\exists x{:}\ Mx \wedge (\exists y{:}\ By)\ Cxy)\ Axd$$

A man, who is carrying a box, is at the door
A man is such that (he is carrying a box and he is at the door)
(∃x: x *is a man*) x *is carrying a box and* x *is at the door*
(∃x: Mx) (x *is carrying a box* ∧ x *is at the door*)
(∃x: Mx) (*a box is such that (*x *is carrying it)* ∧ Axd)
(∃x: Mx) ((∃y: y *is a box*) x *is carrying* y ∧ Axd)

$$(\exists x{:}\ Mx)\ ((\exists y{:}\ By)\ Cxy \wedge Axd)$$

[A: λxy (x *is at* y); C: λxy (x *is carrying* y); M: λx (x *is a man*); d: *the door*]

Moving all the information about the example claimed to exist to the quantified formula would leave us with

$$\exists x\ (Mx \wedge (\exists y\!: By)\ Cxy \wedge Axd)$$

if we ignore the grouping of the conjuncts. This might be expressed in English as *There is a man, who is carrying a box and is at the door*. The corresponding sentence with a restrictive relative clause, *There is a man who is carrying a box and is at the door*, would say the same thing, but it would be more naturally expressed by stating the various properties of the example in a different order—e.g., *There is a man at the door who is carrying a box*.

If we restate the second existential using an unrestricted quantifier, we obtain

$$\exists x\ (Mx \wedge \exists y\ (By \wedge Cxy) \wedge Axd)$$

which is the form of the (slightly awkward) English sentence *There is a man and there is a box he is carrying and he is at the door*, which cannot be analyzed as a conjunction because of the pronouns *he*. While we cannot give the main existential narrower scope than conjunction, it is possible to give the second existential wider scope and write (after some regrouping of conjuncts)

$$\exists x\ \exists y\ ((Mx \wedge By) \wedge (Cxy \wedge Axd))$$

This can be thought of as the analysis of *There is a man and a box and the man is carrying the box and is at the door*, where *the man* and *the box* serve to mark cross reference, or of the analogous sentence *There is a man and a box and he is carrying it and is at the door*, where we use ordinary pronouns instead.

This last form claims the existence of a pair of objects exemplifying the relation λxy (x *is a man and* y *is a box and* x *is carrying* y *and is at the door*). That comes to the same thing as claiming the existence of a man and box which exemplify the relation λxy (x *is carrying* y *and is at the door*), something that can be expressed symbolically by using a pair of restricted quantifiers:

$$(\exists x\!: Mx)\ (\exists y\!: By)\ (Cxy \wedge Axd)$$

This may have no very natural English rendering but it can be expressed by *Some man and box are such that he is carrying it and is at the door*.

The form of restatement used in the last two cases—that is, expanding the scope of an existential to include the whole of a conjunction when it will bind no variables in the other conjuncts—is always possible. And, of course, the opposite operation—narrowing the scope of an existential to the conjuncts of a conjunction in which it actually binds variables—is equally legitimate. Looked at from the latter point of view, the following equivalences (where ϕ has no free occurrence of x and $(\exists x\ldots)$ is either a restricted or unrestricted quantifier)

$$(\exists x\ldots)\ (\phi \wedge \theta x) \Leftrightarrow \phi \wedge (\exists x\ldots)\ \theta x$$
$$(\exists x\ldots)\ (\theta x \wedge \phi) \Leftrightarrow (\exists x\ldots)\ \theta x \wedge \phi$$

can be described as ***confinement principles***, as can obversion.

The change between universal to existential along with confinement in obversion is the exception rather than the rule; confinement of unrestricted quantifiers (both existential and universal) is possible in most other cases following the lines shown above. (On the other hand, it is not always possible to confine the scope of restricted quantifiers to components in which they bind variables. For example, $(\forall x\colon \rho x)\ (\phi \wedge \theta x)$ is not equivalent to $\phi \wedge (\forall x\colon \rho x)\ \theta x$; the first is true and the second is false in a case where ϕ is false but the extension of ρ is empty, for then there can be no counterexample to a generalization over than extension.)

Apart from negations, the only locus of confinement that forces a change between universals and existentials is the antecedent of a conditional. That is also a location where *any* can be used in contrast with *every*, and one of the forms of confinement declares the equivalence of the two natural analyses of such a sentence. Here are the two approaches in the case of an example from 7.3.3

If anyone backs out, the trip will be canceled
Everyone is such that (if he or she backs out, the trip will be canceled)
(∀x: x *is a person*) (*if* x *backs out, the trip will be canceled*)
(∀x: Px) (x *will back out* → *the trip* *will be canceled*)

$$(\forall x\colon Px)\ (Bx \rightarrow Ct)$$
$$\forall x\ (Px \rightarrow (Bx \rightarrow Ct))$$

If anyone backs out, the trip will be canceled
Someone will back out → *the trip will be canceled*
Someone is such that (he or she will back out) → *the trip* *will be canceled*
(∃x: x *is a person*) x *will back out* → Ct

$$(\exists x\colon Px)\ Bx \rightarrow Ct$$
$$\exists x\ (Px \wedge Bx) \rightarrow Ct$$

[B: λx (x *will back out*); C: λx (x *will be canceled*); P: λx (x *is a person*); t: *the trip*]

This examples illustrates the following general confinement principle (where, as before, ϕ must contain no free occurrences of x):

$$(\forall x\ldots)\ (\theta x \rightarrow \phi) \Leftrightarrow (\exists x\ldots)\ \theta x \rightarrow \phi$$

Note that this principle concerns only cases where variables bound by the quantifier are limited to the antecedent of a conditional; confinement to the consequent of a conditional follows the same pattern as confinement to a component of a conjunction or disjunction. The principle is also limited to cases where the quantifiers with wide scope is universal; an unrestricted existential with wide scope can be confined to the antecedent of a conditional (provided it is changed to a universal)

$$\exists x\ (\theta x \rightarrow \phi) \Leftrightarrow \forall x\ \theta x \rightarrow \phi$$

but a restricted existential cannot be confined in this way without weakening the claim being made.

The analogies between restricted universals and conditionals and the possibility of a contrast between *any* and *every* when a quantifier phrase appears within the quantifier phrase of a generalization should suggest that a confinement principle might hold also in such a case. A principle of this sort is illustrated by the following two equivalent analyses of an example from 7.4.2 :

Everything that is relevant to anything is worth knowing
Everything is such that (everything that is relevant to it is worth knowing)
∀x *everything that is relevant to* x *is worth knowing*
∀x *everything that is relevant to* x *is such that (it is worth knowing)*
∀x (∀y: y *is relevant to* x) y *is worth knowing*

∀x (∀y: Ryx) Wy
∀x ∀y (Ryx → Wy)

Everything that is relevant to anything is worth knowing
Everything that is relevant to something is such that (it is worth knowing)
(∀y: y *is relevant to something*) y *is worth knowing*
(∀y: *something is such that (*y *is relevant to it)*) y *is worth knowing*
(∀y: ∃x y *is relevant to it* x) Wy

(∀y: ∃x Ryx) Wy
∀y (∃x Ryx → Wy)

[R: λxy (x *is relevant to* y); W: λx (x *is worth knowing*)]

In this case, the general confinement principle takes the form

$$(\forall x\ldots)\ (\forall y\!:\ \rho xy)\ \theta y \Leftrightarrow (\forall y\!:\ (\exists x\ldots)\ \rho xy)\ \theta y$$

where the formula ρxy may contain free occurrences of the variable x as well as y but θy may not contain free occurrences of x (and any restriction on the quantifiers (∀x...) and (∃x...) may not contain free occurrences of y).

Similarly, confinement of an existential within the restricting formula of an existential is possible when all the variables it binds are in that formula (and requires no change to a universal). Indeed, we might regard *A man who is carrying a box is at the door* as the result of applying such a principle to *A box is such that some man who is carrying it is at the door*.

On the other hand, there is no analogous principle for an existential that binds variables only in the restriction of a universal because confining such a quantifier would involve reversing the relative scope of an existential and a universal and could alter meaning in ways to be discussed in 8.2. And there is also no general principle for confining universals to existentials under similar conditions.