## 8.1.2. Obversion

Just as generalizations deny the existence of counterexamples, denials of generalizations claim the existence of such examples. This suggests that it should be possible to restate the denials of generalizations as existential claims. And it is not hard to see how. For example, *Not every dog barks* claims the existence of an example among dogs of something that does not bark, so it is equivalent to *Some dog does not bark* (or *Some dogs do not bark*). And *Not only trucks were advertised* claims the existence of a non-truck that was advertised, so it is equivalent to *Some non-truck was advertised*. The general principle behind these equivalences takes the form

$$\neg\ (\forall x\text{: } \rho x)\ \theta x \Leftrightarrow (\exists x\text{: } \rho x)\ \overline{\theta x}$$

To deny that the predicate θ is true generally of the extension of ρ (which is what $\neg\ (\forall x\text{: } \rho x)\ \theta x$ does) is to claim the existence, in the extension of ρ, of a counterexample—i.e., an object of which the predicate $\lambda x\ \overline{\theta x}$ is true. And this is just what $(\exists x\text{: } \rho x)\ \overline{\theta x}$ claims. This is one form of a principle for which we will adapt the traditional term ***obversion***. (This term is usually applied more narrowly to equivalences where the generalization is direct and where the negation is part of a noun phrase in one of the two equivalent sentences—each of our examples fails on one of these scores.) The bar notation functions here as before to provide a single notation for both adding and removing negation. With it, the principle says that the denial of a negative generalization is equivalent to a claim of exemplification for either a doubly negative or an affirmative predicate. The sentence *Not everyone failed to laugh* is equivalent to *Someone laughed* as well as to *Someone did not fail to laugh*.

A second form of obversion can be found in the possibility of using a generalization to deny an existential claim. To deny *Some dog climbs trees*, we can assert *No dog climbs trees*. And, in general, to deny the existence of an example, we can make an appropriate negative generalization:

$$\neg\ (\exists x\text{: } \rho x)\ \theta x \Leftrightarrow (\forall x\text{: } \rho x)\ \overline{\theta x}$$

The two forms of obversion for restricted quantifiers are matched by two forms for unrestricted quantifiers and we can use some notation introduced in 7.3.2 to state the principles for both sorts of quantifiers at once:

$$\neg (\forall x\ldots)\ \theta x \Leftrightarrow (\exists x\ldots)\ \overline{\theta x}$$
$$\neg (\exists x\ldots)\ \theta x \Leftrightarrow (\forall x\ldots)\ \overline{\theta x}$$

That is, to deny that a predicate is universal is to say that its negation is exemplified; and to deny that a predicate is exemplified is to say that its negation is universal.

The second form of obversion shows the equivalence of the two sorts of analysis that we can now give for many uses of *any* (when it contrasts with *every*). The following repeats and extends an example of 7.3.3 :

*Tom didn't see anything*
*Everything is such that (Tom didn't see it)*
∀x (*Tom didn't see* x)
∀x ¬ Tom saw x

∀x ¬ Stx

*Tom didn't see anything*
¬ *Tom saw something*
¬ *something is such that (Tom saw it)*
¬ ∃x Tom saw x

¬ ∃x Stx

[S: λxy (x *saw* y); t: *Tom*]

These two symbolic forms are often equally close to the forms of English sentences. Although negated existentials preferable to negative generalizations for the purposes of the exercises in this chapter, the role of negative generalizations in deductive reasoning is clearer both intuitively and in the context of the system of derivations we will use.