8.1.1. Exemplification

Although we have looked at quantification and quantifiers, the idea of quantity has not been much in evidence. Of course it could be found in discussions of generalization if we look hard enough because any generalization can be understood to claim that its counterexamples number 0. This way of looking at generalizations is rather forced, but the sorts of claims we will now consider can all be stated rather naturally by reference to numbers.

Our study will have a somewhat different character in another respect, too. We had to devote much effort to analyzing generalizations before we could put them into symbolic form, but once that analysis was carried out, the symbolic forms were easily stated. In this chapter, our symbolic analyses will require much less preparatory work on the English sentences. This is in part because we can carry over ideas from the last chapter, but it is in large part due to the relative simplicity of the means of expression we will encounter in English. However, before long, we will have considered quite a variety of numerical claims. Since most of these will be expressed using only one new symbol, we have to devote more of our attention to developing the symbolic means to represent English forms. Thus the focus of our attention will shift slightly, though noticeably, from English to the symbolic language.

The first evidence of this is that we will begin our discussion of our first new sort of logical form by considering its symbolic version. The ***unrestricted existential quantifier*** is an operator that applies to a one-place predicate abstract, its ***quantified predicate***, to say that the extension of the predicate contains at least one value, that it is ***non-empty***. We will use the sign ∃ (named ***there exists***) for this operation. A sentence ∃[λx θx] formed using this quantifier says that the predicate λx θx is ***exemplified***, that there is some value (in the range **R**) that serves as an example of a thing that λx θx is true of. Thus the sentence *Something fell* could be represented as ∃[λx Fx] (using F: λx (x *fell*)).

The ***restricted existential quantifier*** is used to claim the existence of examples that are not merely in the referential range but in some more specific class. It applies to a pair of one-place predicates, its ***restricting*** and ***quantified*** predicates, to form a sentence ∃[λx ρx][λx θx] that asserts that the extension of θ contains at least one member of the extension of ρ. So *Some dog climbs trees* could be represented as ∃[λx Dx][λx Cx] (using D: λx (x *is a dog*); C: λx (x *climbs trees*)).

We will generally use abbreviations that, like those used for the universal quantifiers, suppress the symbol λ. Thus $\exists\ [\lambda x\ \theta x]$ becomes $\exists x\ \theta x$, and $\exists[\lambda x\ \rho x]\ [\lambda x\ \theta x]$ becomes $(\exists x{:}\ \rho x)\ \theta x$. The two examples above could be written in this way as $\exists x\ Fx$ and $(\exists x{:}\ Dx)\ Cx$, respectively. We will continue to refer to the component formulas ρx and θx as the ***restricting*** and ***quantified*** formulas, respectively.

As with universals, we have principles of equivalence that enable us to restate restricted existentials as unrestricted existentials, and vice versa.

$$(\exists x{:}\ \rho x)\ \theta x \Leftrightarrow \exists x\ (\rho x \wedge \theta x)$$
$$\exists x\ \theta x \Leftrightarrow (\exists x{:}\ x = x)\ \theta x$$

These should be compared to the analogous principles for the universal quantifiers discussed in 7.2.1. The only disanalogy appears in the first, which contains a conjunction at a point where the corresponding principle for universals contains a conditional.

The reason is this. While the restricting predicate serves with both universals and existentials to make the claim more specific or less general, this has a different effect on the strength of the claim—on how much is said—in the two cases. When a generalization is restricted, it generalizes about a more narrowly specified class, and its claim is weakened; it says less, and this is represented by the hedging effect of the conditional. On the other hand, when an existential claim is restricted, the kind of example claimed to exist is more fully specified and the claim is strengthened; it says more, and this is represented by the strengthening effect of conjunction.

In both of the English examples above, the quantifier phrases we analyzed had *some* as their quantifier word. This is not the only word that can signal the presence of an existential quantifier. In particular, as was discussed in 7.3.1, one of the chief uses of the indefinite article *a* is to claim the existence of an example, to make an ***existential claim*** or ***claim of exemplification***. Thus either *Some dog barked* or *A dog barked* could be used in English to express the existential claim represented symbolically by $(\exists x{:}\ Dx)\ Bx$ (using B: λx (x *barked*); D: λx (x *is a dog*)).

Although there is more than one way of expressing an existential claim, we do not have several kinds of existential claim in the way in which we have several kinds of generalization. That is, there is no quantifier word that indicates that the denial of the quantified predicate is being exemplified and none that indicates that the example is to be found outside the class picked out by the class indicator. At least, this is so if we follow the policy of 7.3.1 and analyze *not every* and *not only* rather than treating them as units. Of course, existential quantifiers can apply to negative predicates; but the corresponding English forms will be like our symbolic notation in having such negation as an explicit part of the quantified predicate or the class indicator instead of signaling the presence of negation by the quantifier word used.

There is one special problem concerning existential claims that deserves some discussion though it cannot be given a fully satisfactory treatment here. The word *some* is often used with plural noun phrases, as in *Some mice were in the attic*, and bare plural common noun phrases are sometime used to the same effect, as in *Mice were in the attic*. One would expect such sentences to claim the existence of multiple examples, but if we consider their implications rather than their implicatures, this does not seem to be so. Suppose you knew that one and only one mouse had been in the attic. If you were asked the question *Were mice in the attic?* the natural response would be *Yes, one was* rather than *No, only one was*. This suggests that we are prepared to count a sentence like *Mice were in the attic* as true even when there is only one example—although it would generally be misleading to assert it under such conditions.

There is another argument for the same conclusion. Under one interpretation of it, the ambiguous sentence *Mice were not in the attic* is the denial of *Mice were in the attic*. And, so understood, it is equivalent to *No mice were in the attic*. But *No mice were in the attic* and *No mouse was in the attic* are both negative generalizations that make the same claim: that there is no example to be found among mice of a thing that was in the attic. The moral is that the distinction between singular and plural in English escapes our analysis. This is not to say that we have no way to represent claims that actually *imply* the existence of multiple examples; we will encounter quite a variety beginning in 8.3.2.