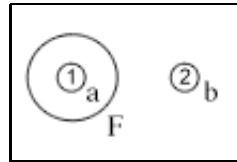


7.7.xa. Exercise answers

These answers handle restricted quantifiers using the rules RUG, SB, SC, and MCR rather than RUP and RUC; see [7.6.xa](#) for advice for converting them into derivations that instead use the latter rules along with rules for conditionals.

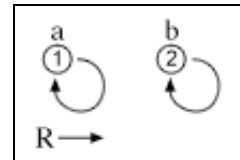
1.

	Fa		
	ⓑ	¬ Fb	
		○	Fa, ¬ Fb ⇒ ⊥
		⊥	2
2 IP	Fb		1
1 UG	∀x Fx		



2.

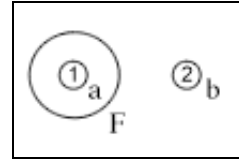
	∀x Rxx	a:1, b:3	
1 UI	Raa		
3 UI	ⓑ	Rbb	
		¬ Rba	
		○	Raa, Rbb, ¬ Rba ⇒ ⊥
		⊥	4
4 IP	Rba		2
2 UG	∀x Rxa		



3.

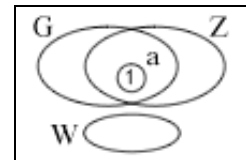
	∀x ¬ Fx	a:3
	∀x Fx	a:2
2 UI	Fa	(4)
3 UI	¬ Fa	(4)
	•	
4 Nc	⊥	1
1 RAA	¬ ∀x Fx	

	$\neg \forall x Fx$	
	(a)	
	Fa	
	(b)	
	$\neg Fb$	
	○	$Fa, \neg Fb \Rightarrow \perp$
	\perp	5
5 IP	Fb	4
4 UG	$\forall x Fx$	3
3 CR	\perp	2
2 RAA	$\neg Fa$	1
1 UG	$\forall x \neg Fx$	



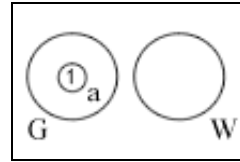
4.

	$(\forall x: Wx) \neg Gx$	a:4
	$(\forall x: Zx) \neg Wx$	a:2
	(a)	
	Za	(2)
	$\neg Wa$	
2 SB	Ga	(4)
	$\neg Wa$	
	○	$Za, \neg Wa, Ga \Rightarrow \perp$
	\perp	3
4 SC	$\neg Ga$	
3 RAA	$\neg Ga$	
1 RUG	$(\forall x: Zx) \neg Gx$	

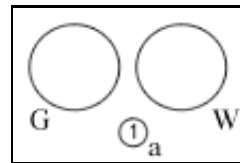


5.

	$(\forall x: Wx) \neg Gx$	a:2
	$(\forall x: Wx) Gx$	a:4,a:6
5 IP	$\neg Wa$	
	$\neg Wa$	
	\circ	$\neg Wa \not\Rightarrow \perp$
	\perp	5
	Wa	4
4 MCR	Ga	
	\circ	$\neg Wa, Ga \not\Rightarrow \perp$
	\perp	4
	\perp	3
3 IP	Wa	2
6 SC	$\neg Ga$	(6)
	$\neg Wa$	
	\circ	$\neg Wa, \neg Ga \not\Rightarrow \perp$
	\perp	2
2 MCR	\perp	1
1 RAA	$\neg (\forall x: Wx) Gx$	

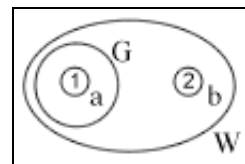


divides the 1st and 2nd gaps



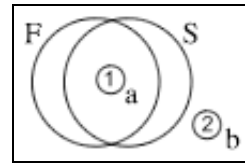
divides the 1st and 3rd gaps

	$\neg (\forall x: Wx) Gx$	
5 IP	$\textcircled{a} Wa$	
	Ga	
	$\textcircled{b} Wb$	
	$\neg Gb$	
	\circ	$Wa, Ga, Wb, \neg Gb \not\Rightarrow \perp$
	\perp	5
4 RUG	Gb	4
3 CR	$(\forall x: Wx) Gx$	3
	\perp	2
2 RAA	$\neg Ga$	1
1 RUG	$(\forall x: Wx) \neg Gx$	



6.

	$\forall x (Fx \vee \neg Sx)$	a:4, b:9
	$\neg \forall x Fx$	
	ⓐ	
	Sa	(5)
4 UI	Fa \vee \neg Sa	5
5 MTP	Fa	
	ⓑ	
	$\neg Fb$	(10)
9 UI	Fb \vee $\neg Sb$	10
10 MTP	$\neg Sb$	
	○	Sa, Fa, $\neg Fb$, $\neg Sb \Rightarrow \perp$
	\perp	8
8 IP	Fb	7
7 UG	$\forall x Fx$	6
6 CR	\perp	3
3 RAA	$\neg Sa$	2
2 UG	$\forall y \neg Sy$	1
1 PE	$\forall x Fx \vee \forall y \neg Sy$	



7.

	$\neg \forall x \neg \forall y Rxy$	
	ⓐ	
	$\forall y \neg Ray$	a:3, b:6
3 UI	$\neg Raa$	
	ⓑ	
6 UI	$\neg Rab$	
	$\forall y Rby$	a:8, b:9
8 UI	Rba	
9 UI	Rbb	
	○	$\neg Raa, \neg Rab, Rba, Rbb \Rightarrow \perp$
	\perp	7
7 RAA	$\neg \forall y Rby$	5
5 UG	$\forall x \neg \forall y Rxy$	4
4 CR	\perp	2
2 RAA	$\neg \forall y \neg Ray$	1
1 UG	$\forall x \neg \forall y \neg Rxy$	

