

7.7.2. Soundness

An utterly sound rule, in the sense introduced in [2.3.2](#), does not throw away gap-dividing interpretations as it develops a gap. That is, any structure dividing a gap to which the rule is applied will divide at least one child gap produced by the rule. In applying this idea to the rules for universals, we are faced with a problem caused by rules that introduce new vocabulary. New vocabulary is introduced always by the planning rules UG and RUG, which introduce parameters, and it must be introduced by the four exploitation rules if generalization would otherwise go unexploited. A structure that divides a gap before a rule is applied may fail to divide a gap afterwards simply because it gives no interpretation at all to new vocabulary that the rule introduces. And, even if it does happen to interpret this new vocabulary, the interpretations it gives have played no role in dividing the gap before the vocabulary was introduced, so we may need to revise them as we go on. In short, if an interpretation dividing a gap is to divide any of its children, we may need to provide new interpretations of new vocabulary appearing in that child.

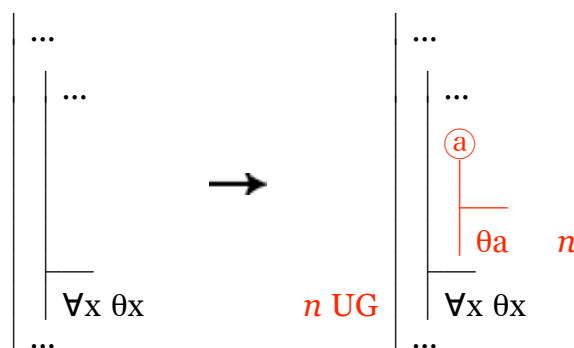
To begin to handle this problem, let us first be more explicit about the conditions under which a structure counts as an interpretation of a gap. In previous chapters we took it for granted that the interpretations we considered interpreted all vocabulary appearing anywhere in the derivation since all such vocabulary appeared in the initial premises and conclusion and we wanted all our interpretations to give truth values to these sentences. Now we need to be more flexible, so let us say that a structure *interprets* a gap if it assigns interpretations to all the non-logical vocabulary that appears in resources or goals of the gap or any of its ancestors. Such a structure must interpret all vocabulary in the initial premises and conclusion of the ultimate argument of the derivation and also interpret all parameters introduced along the way to the gap in question, but it need not interpret parameters whose occurrences are boxed off from the gap we are considering. Notice that we allow an interpretation of a gap to provide interpretations of vocabulary not appearing in a gap. This means that any interpretation of gap not only interprets the vocabulary of all its ancestor gaps but in fact counts as an interpretation of those gaps. Among the structures interpret a gap, we distinguish those that divide it in the same way we have in the past—that is, as the structures that make its active resources true and its goal false.

In order to adapt the definition of soundness to the possibility of changing vocabulary, we can no longer require that, when an

interpretation divides a gap, an identical interpretation divides at least one child since we may need to extend or modify the interpretation to accommodate new vocabulary. Let us say that two interpretations **agree for** a gap when they have the same referential range and give the same interpretation to all vocabulary appearing in the gap and all its ancestors. This idea is motivated by a principle concerning structures that should seem plausible but that we will not argue for: if two structures have the same range and agree on the interpretation of all vocabulary in a sentence, then they each assign the same truth value to that sentence. It follows that if two interpretations agree for a gap, then one will divide the gap if and only if the other does (and this will be true also for all ancestors of the gap).

Given these ideas, we will say that a rule is **utterly sound** when, for any interpretation dividing a gap before the rule is applied, we can find an interpretation that agrees with the first for that gap and that divides at least one child gap resulting from the rule. According to this definition an utterly sound rule need not preserve gap-dividing structures unchanged; it must preserve what was essential to the function of such a structure in dividing a parent gap but it may force it to be elaborated or altered in order to interpret a gap resulting from the rule. We will say that a rule is **minimally sound** when it preserves (in this way) interpretations that divide both the gap to which the rule is applied and all of its ancestors. Equivalently, a rule is minimally sound when for any interpretation dividing a path before the rule is applied to its final gap, we can find an interpretation agreeing with it on this final gap that divides at least one path that results from applying the rule.

The rules UG, RUG, UI, and MCR are all utterly sound in this new sense. (The rules SB and SC would be utterly sound if we applied them only to active resources but we allow the resource they do not exploit to be only available and, because of that, they are only minimally sound.) The actual arguments showing utter soundness are not very surprising, so we will look at only the case of UG.



Suppose S is a structure dividing the gap on the left. Since S makes the goal $\forall x \theta x$ false, it must assign θ an extension that does not include the whole referential range. Let S' be like S except in assigning to the parameter a some value outside the extension of θ . Then S' will agree with S for the gap at the left (since a does not appear before UG is applied), and it will make θa false. So S' (like S) will make all active resources of the two gaps true, and it will make the new goal false (whether or not S does). So, given a structure S dividing the old gap, the essentials of the way it does so are preserved in a structure S' dividing the new one; and that means that UG is utterly sound.

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