

## 7.6.xa. Exercise answers

These answers use the rules RUG, SB, SC, and MCR rather than RUP and RUC. Derivations using the latter rules can be constructed from them by replacing each use of one of the first four rules by a series of three steps as shown in the following table:

<i>basic rule</i>	<i>alternative approach using RUP and RUC</i>
RUG	RUC, UG, CP
SB	RUP, UI, MPP
SC	RUP, UI, MTT
MCR	RUP, UI, RC

1. a.

	$\forall x \forall y Rxy$	a:1
	$(\forall x: Rxx) Gx$	a:3
1 UI	$\forall y Ray$	a:2
2 UI	$Raa$	(3)
3 SB	$Ga$	(4)
	•	
4 QED	$Ga$	

b.

	$(\forall x: Fx) Gx$	a:3		$\forall x (Fx \rightarrow Gx)$	a:2
	(a)			(a)	
	$Fa$	(3)		$Fa$	(3)
3 SB	$Ga$	(4)	2 UI	$Fa \rightarrow Ga$	3
	•		3 MPP	$Ga$	(4)
4 QED	$Ga$	2		•	
2 CP	$Fa \rightarrow Ga$	1	4 QED	$Ga$	1
1 UG	$\forall x (Fx \rightarrow Gx)$		1 RUG	$(\forall x: Fx) Gx$	

c.

	$Fa$	(2)		$(\forall x: x = a) Fx$	a:2
	(b)			$\neg Fa$	(2)
	$b = a$	a-b		$\neg a = a$	(3)
	•		2 SC	•	
2 QED=	$Fb$	1	3 DC	$\perp$	1
1 RUG	$(\forall x: x = a) Fx$		1 IP	$Fa$	

d.

	$\forall x \forall y (Rxy \rightarrow \neg Ryx)$	a:5
	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">(a)</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">(b)</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\neg a = b</math> </div> </div> </div> </div>	
	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">4</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rab \wedge Rba</math> </div> <div style="margin-left: 10px;">4</div> </div>	
4 Ext	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">4</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rab</math> </div> <div style="margin-left: 10px;">(7)</div> </div>	
4 Ext	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">4</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rba</math> </div> <div style="margin-left: 10px;">(8)</div> </div>	
5 UI	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">5</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\forall y (Ray \rightarrow \neg Rya)</math> </div> <div style="margin-left: 10px;">b:6</div> </div>	
6 UI	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">6</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rab \rightarrow \neg Rba</math> </div> <div style="margin-left: 10px;">7</div> </div>	
7 MPP	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">7</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\neg Rba</math> </div> <div style="margin-left: 10px;">(8)</div> </div>	
	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">8</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\perp</math> </div> <div style="margin-left: 10px;">3</div> </div>	
8 Nc	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">3</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\neg (Rab \wedge Rba)</math> </div> <div style="margin-left: 10px;">2</div> </div>	
3 RAA	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">2</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>(\forall y: \neg a = y) \neg (Ray \wedge Rya)</math> </div> <div style="margin-left: 10px;">1</div> </div>	
2 RUG	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">1</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\forall x (\forall y: \neg x = y) \neg (Rxy \wedge Ryx)</math> </div> </div>	
1 UG	$\forall x (\forall y: \neg x = y) \neg (Rxy \wedge Ryx)$	

e.

	$\forall x (\forall y: \neg x = y) \neg (Rxy \wedge Ryx)$	a:5
	$\forall x \neg Rxx$	a:8
	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">(a)</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">(b)</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rab</math> </div> <div style="margin-left: 10px;">(6),(9)</div> </div> </div> </div>	
	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">5</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rba</math> </div> <div style="margin-left: 10px;">(6)</div> </div>	
5 UI	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">5</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>(\forall y: \neg a = y) \neg (Ray \wedge Rya)</math> </div> <div style="margin-left: 10px;">b:7</div> </div>	
6 Adj	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">6</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rab \wedge Rba</math> </div> <div style="margin-left: 10px;">X,(7)</div> </div>	
7 SC	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">7</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>a = b</math> </div> <div style="margin-left: 10px;">a=b</div> </div>	
8 UI	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">8</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\neg Raa</math> </div> <div style="margin-left: 10px;">(9)</div> </div>	
	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">9</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\perp</math> </div> <div style="margin-left: 10px;">4</div> </div>	
9 Nc=	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">4</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\neg Rba</math> </div> <div style="margin-left: 10px;">3</div> </div>	
4 RAA	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">3</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Rab \rightarrow \neg Rba</math> </div> <div style="margin-left: 10px;">2</div> </div>	
3 CP	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">2</div> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\forall y (Ray \rightarrow \neg Rya)</math> </div> <div style="margin-left: 10px;">1</div> </div>	
2 UG	$\forall x \forall y (Rxy \rightarrow \neg Ryx)$	
1 UG	$\forall x \forall y (Rxy \rightarrow \neg Ryx)$	

f.

	$(\forall x: Px) (\forall y: Py) (\forall z: Pz \wedge Lzx) Lyz$	b:6,a:10
	⊖ a	
	Pa	(8), (10)
	⊖ b	
	Pb	(6)
	⊖ Lab	
	Lab	(8)
	⊖ c	
	Pc	(11)
	⊖ d	
	Pd	(7), (12)
6 SB	$(\forall y: Py) (\forall z: Pz \wedge Lzb) Lyz$	d:7
7 SB	$(\forall z: Pz \wedge Lzb) Ldz$	a:9
8 Adj	$Pa \wedge Lab$	X, (9)
9 SB	Lda	(12)
10 SB	$(\forall y: Py) (\forall z: Pz \wedge Lza) Lyz$	c:11
11 SB	$(\forall z: Pz \wedge Lza) Lcz$	d:13
12 Adj	$Pd \wedge Lda$	X, (13)
13 SB	Lcd	(14)
	•	
14 QED	Lcd	5
5 RUG	$(\forall w: Pw) Lcw$	4
4 RUG	$(\forall z: Pz) (\forall w: Pw) Lzw$	3
3 CP	$Lab \rightarrow (\forall z: Pz) (\forall w: Pw) Lzw$	2
2 RUG	$(\forall y: Py) (Lay \rightarrow (\forall z: Pz) (\forall w: Pw) Lzw)$	1
1 RUG	$(\forall x: Px) (\forall y: Py) (Lxy \rightarrow (\forall z: Pz) (\forall w: Pw) Lzw)$	

g.

	$\forall x (\forall y: gx = y) Fy$	ha:2
	⊖ a	
2 UI	$(\forall y: g(ha) = y) Fy$	g(ha):4
3 EC	$g(ha) = g(ha)$	X, (4)
4 SB	$F(g(ha))$	(5)
	•	
5 QED	$F(g(ha))$	1
1 UG	$\forall x F(g(hx))$	

**h.**

	$\forall x \forall y Rxy$ $(\forall x: \forall y Ryx) (Fx \rightarrow Gx)$		b:5 a:3
	<div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">a</span> </div>	Fa	(8)
	<div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">b</span> </div> </div> </div>	$\neg Ga$	(9)
5 UI	<div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <math>\forall y Rby</math> </div> </div> </div>	Rba	a:6 (7)
6 UI	<div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <div style="border-left: 1px solid black; padding-left: 5px;"> <div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <span style="display: inline-block; width: 10px; height: 10px; background-color: black; border-radius: 50%;"></span> </div> </div> </div>	Rba	4
7 QED	<div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\forall y Rya</math> </div> </div>	$\forall y Rya$	3
4 UG	<div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>Fa \rightarrow Ga</math> </div> </div>	$Fa \rightarrow Ga$	8
8 MPP	<div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <div style="border-left: 1px solid black; padding-left: 5px;"> <span style="display: inline-block; width: 10px; height: 10px; background-color: black; border-radius: 50%;"></span> </div> </div>	Ga	(9)
9 Nc	<div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\perp</math> </div> </div>	$\perp$	3
3 MCR	<div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <div style="border-left: 1px solid black; padding-left: 5px;"> <math>\perp</math> </div> </div>	$\perp$	2
2 IP	<div style="border-bottom: 1px solid black; padding-bottom: 2px;"> <div style="border-left: 1px solid black; padding-left: 5px;"> Ga </div> </div>	Ga	1
1 RUG	$(\forall x: Fx) Gx$		

i.

	$(\forall x: Rax) Sax$	c:9
	$Pa \wedge \forall x \neg Sax$	1
	$(\forall x: Px \wedge \forall y \neg Rxy) \forall z Fxz$	a:4
1 Ext	$Pa$	(6)
1 Ext	$\forall x \neg Sax$	c:8
	(b)	
	$\neg Fab$	
	$\bullet$	
6 QED	$Pa$	5
	(c)	
8 UI	$\neg Sac$	(9)
9 SC	$\neg Rac$	(10)
	$\bullet$	
10 QED	$\neg Rac$	7
7 UG	$\forall y \neg Ray$	5
5 Cnj	$Pa \wedge \forall y \neg Ray$	4
	$\forall z Faz$	b:9
	$Fab$	
	$\bullet$	
	$\perp$	4
4 MCR	$\perp$	3
3 IP	$Fab$	2
2 UG	$\forall z Faz$	

2. a. *Every road sign was colored*  
*Every stop sign was a road sign*  
*If anything was colored, it was painted*  
*Every stop sign was painted*

	$(\forall x: Dx) Cx$	a:3
	$(\forall x: Sx) Dx$	a:2
	$\forall x (Cx \rightarrow Px)$	a:4
	(a)	
	$Sa$	(2)
2 SB	$Da$	(3)
3 SB	$Ca$	(5)
4 UI	$Ca \rightarrow Pa$	
5 MPP	$Pa$	(6)
	$\bullet$	
6 QED	$Pa$	1
1 RUG	$(\forall x: Sx) Px$	

- b.** *No road sign was colored*  
*Every stop sign was a road sign*  
*If anything was red, it was colored*  
*No stop sign was red*

	$(\forall x: Dx) \neg Cx$	a:3
	$(\forall x: Sx) Dx$	a:2
	$\forall x (Rx \rightarrow Cx)$	a:4
	(a)	
	Sa	(2)
2 SB	Da	(3)
3 SB	$\neg Ca$	(5)
4 UI	$Ra \rightarrow Ca$	5
5 MTT	$\neg Ra$	(6)
	•	
6 QED	$\neg Ra$	1
1 RUG	$(\forall x: Sx) \neg Rx$	

- c.** *Only road signs were colored*  
*Every road sign was a traffic marker*  
*If anything was red, it was colored*  
*Only traffic markers were red*

	$(\forall x: \neg Dx) \neg Cx$	a:3
	$(\forall x: Dx) Mx$	a:2
	$\forall x (Rx \rightarrow Cx)$	a:4
	(a)	
	$\neg Ma$	(2)
2 SC	$\neg Da$	(3)
3 SB	$\neg Ca$	(5)
4 UI	$Ra \rightarrow Ca$	5
5 MTT	$\neg Ra$	(6)
	•	
6 QED	$\neg Ra$	1
1 RUG	$(\forall x: \neg Mx) \neg Rx$	

- d. *Among road signs, all except colored ones were replaced*  
*Every stop sign was a road sign*  
*If anything was colored, it was painted*  


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*Among stop signs, all except painted ones were replaced*

	$(\forall x: Dx \wedge \neg Cx) Lx$	a:7
	$(\forall x: Sx) Dx$	a:3
	$\forall x (Cx \rightarrow Px)$	a:4
	(a)	
	Sa $\wedge$ $\neg$ Pa	2
2 Ext	Sa	(3)
2 Ext	$\neg$ Pa	(5)
3 SB	Da	(6)
4 UI	$Ca \rightarrow Pa$	5
5 MTT	$\neg Ca$	(6)
6 Adj	Da $\wedge$ $\neg Ca$	X, (7)
7 SB	La	(8)
	•	
8 QED	La	1
1 RUG	$(\forall x: Sx \wedge \neg Px) Lx$	

- e. *Everyone watched every snake*  
*Every cobra is a snake*  


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*Everyone watched every cobra*

	$(\forall x: Px) (\forall y: Sy) Wxy$	a:3
	$(\forall x: Cx) Sx$	b:4
	(a)	
	Pa	(3)
	(b)	
	Cb	(4)
3 SB	$(\forall y: Sy) Way$	b:5
4 SB	Sb	(5)
5 SB	Wab	(6)
	•	
6 QED	Wab	2
2 RUG	$(\forall y: Cy) Way$	1
1 RUG	$(\forall x: Px) (\forall y: Cy) Wxy$	

- f. *No one watched every snake*  
*Every snake is a reptile*  


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*No one watched every reptile*

	$(\forall x: Px) \neg (\forall y: Sy) Wxy$	a:2
	$(\forall x: Sx) Rx$	b:6
	(a)	
	Pa	(2)
2 SB	$\neg (\forall y: Sy) Way$	4
	$(\forall y: Ry) Way$	b:7
	(b)	
	Sb	(6)
6 SB	Rb	(7)
7 SB	Wab	(8)
	•	
8 QED	Wab	5
5 RUG	$(\forall y: Sy) Way$	4
4 CR	$\perp$	3
3 RAA	$\neg (\forall y: Ry) Way$	1
1 RUG	$(\forall x: Px) \neg (\forall y: Ry) Wxy$	

- g. *No one watched any snake*  
*Every cobra is a snake*  


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*No one watched any cobra*

	$(\forall y: Sy) (\forall x: Px) \neg Wxy$	a:3
	$(\forall x: Cx) Sx$	a:2
	(a)	
	Ca	(2)
2 SB	Sa	(3)
3 SB	$(\forall x: Px) \neg Wxa$	(4)
	•	
4 QED	$(\forall x: Px) \neg Wxa$	1
1 RUG	$(\forall y: Cy) (\forall x: Px) \neg Wxy$	



- h.** *Everyone who likes every snake was pleased*  
*Every snake is a reptile*  


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*Everyone who likes every reptile was pleased*

	$(\forall x: Px \wedge (\forall y: Sy) Lxy) Dx$	a:4
	$(\forall x: Sx) Rx$	b:8
	ⓐ	
	Pa $\wedge$ $(\forall y: Ry) Lay$	2
2 Ext	Pa	(5)
2 Ext	$(\forall y: Ry) Lay$	b:9
	$\neg Da$	(4)
4 SC	$\neg (Pa \wedge (\forall y: Sy) Lay)$	5
5 MPT	$\neg (\forall y: Sy) Lay$	6
	ⓑ	
	Sb	(8)
8 SB	Rb	(9)
9 SB	Lab	(10)
	•	
10 QED	Lab	7
7 RUG	$(\forall y: Sy) Lay$	6
6 RC	$\perp$	3
3 IP	Da	1
1 RUG	$(\forall x: Px \wedge (\forall y: Ry) Lxy) Dx$	

- i.** *Everyone who likes a snake was pleased*  
*Every cobra is a snake*  


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*Everyone who likes a cobra was pleased*

	$(\forall x: Sx) (\forall y: Py \wedge Lyx) Dy$	a:3
	$(\forall x: Cx) Sx$	a:2
	ⓐ	
	Ca	(2)
2 SB	Sa	(3)
3 SB	$(\forall y: Py \wedge Lya) Dy$	(4)
	•	
4 QED	$(\forall y: Py \wedge Lya) Dy$	1
1 RUG	$(\forall x: Cx) (\forall y: Py \wedge Lyx) Dy$	