7.6.2. Derivations for restricted universals

The rules for restricted quantifiers will resemble those for conditionals as well as those for restricted universals. Figure 7.6.2-1 shows the planning rule, ***Restricted Universal Generalization*** (RUG).

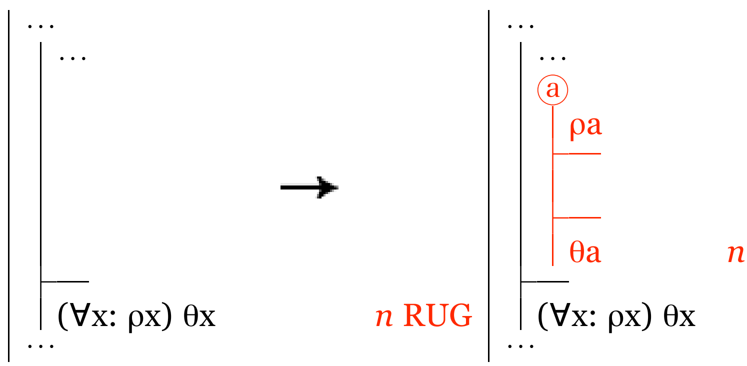

Fig. 7.6.2-1. Developing a derivation at stage *n* by planning for a restricted universal.

The notation (∀x: ρx) θx is more convenient here than (∀x: ...x...) ---x--- but remember that an expression like ρx has the same significance as one like ...x...; that is, it stands for a formula, with the variable x standing for all free occurrences of that variable in it. And the expression ρa stands for the result of putting the parameter a in place of all free occurrences of x in ρx.

As with the planning rule for an unrestricted universal, the parameter a must be new to the derivation. Notice that the general argument begins with a supposition. This is like beginning a general argument concerning triangles by saying, "Suppose *ABC* is a triangle. . . ," so it recognizes one more function for suppositions: to place restrictions on the arbitrary choice of a reference value for a parametric term in an argument that is both general and hypothetical. It is the hypothetical aspect of the argument that forces its conclusion to be restricted: if we have supposed that *ABC* is a triangle, we cannot claim that a property we establish for *ABC* holds of everything, only that it holds of all triangles.

The principles singular Barbara and singular Camestres bear enough similarities to the idea detachment introduced in 4.3.1 that we have extended that term to them. However, the principles for universals do have one difference from the principles for truth-functional compounds: their conclusions do not imply either premise. What they represent is really a sort of partial detachment since the conclusion in each case does imply the conditioned instance of (∀x: ρx) θx for the term τ. This is enough for them to justify a partial exploitation of universals, and that is the most we could expect.

Figures 7.6.2-2 and 7.6.2-3 show rules—named ***Singular Barbara*** (SB) and ***Singular Camestres*** (SC)—that are based on these principles.

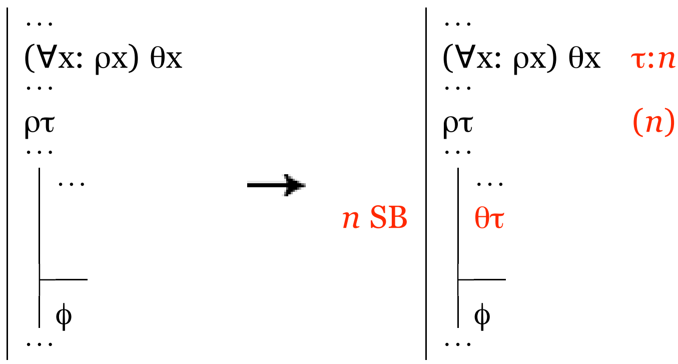

Fig. 7.6.2-2. Developing a derivation at stage *n* by exploiting for a term τ a restricted universal whose restricting predicate is applied to τ in an active resource.

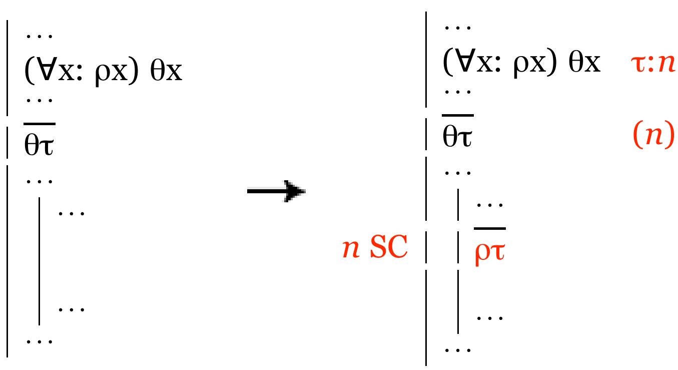

Fig. 7.6.2-3. Developing a derivation at stage *n* by exploiting for a term τ a restricted universal whose quantified predicate is denied of τ in an active resource.

These two are the rules that we will use most often to exploit restricted universals. The fullest use of the first will come by using attachment rules, for the sentence ρτ may be a complex sentence that is entailed by our resources even though it does not appear among them.

The derivation shown below combines these rules with planning rules for both sorts of universal. It establishes the following entailment:

All dogs are mammals ⇒ *If anything affects only dogs, then it affects only mammals*

	(∀x: Dx) Mx	b:4
	ⓐ	
	(∀y: ¬ Dy) ¬ Fay	b:5
	ⓑ	
	¬ Mb	(4)
4 SC	¬ Db	(5)
5 SB	¬ Fab	(6)
	●	
6 QED	¬ Fab	3
3 RUG	(∀z: ¬ Mz) ¬ Faz	2
2 CP	(∀y: ¬ Dy) ¬ Fay → (∀z: ¬ Mz) ¬ Faz	1
1 UG	∀x ((∀y: ¬ Dy) ¬ Fxy → (∀z: ¬ Mz) ¬ Fxz)	

In giving a symbolic version of the conclusion, the quantifier phrase *anything* was dealt with first. This made it possible to analyze the body of its quantified predicate as the conditional *if* x *affects only dogs, then* x *affects only mammals* and to analyze the two components further. The derivation begins at stage 1 with planning for the unrestricted universal conclusion. At stage 2 we plan for the new conditional goal and at stage 3 for a restricted universal, introducing a new parameter and supplying a supposition that begins exploitation of the two restricted universal resources in stages 4 and 5. Notice that the argument for ¬ Fab is doubly general. It uses no special assumption about the reference of the term a and assumes about the term b only that it does not refer to a mammal.

The following example shows a typical use of attachment rules with instantiation for restricted universals:

Everything important that was broken was repaired ⇒ If anything broken was important, it was repaired

	(∀x: Ix ∧ Bx) Rx	b:4
	ⓐ	
	Ba	(3)
	Ia	(3)
3 Adj	Ia ∧ Ba	X,(4)
4 SB	Ra	(5)
	•	
5 QED	Ra	2
2 CP	Ia → Ra	1
1 RUG	(∀x: Bx) (Ix → Rx)	

After stage 2, our resources entail that any object referred to by the parameter a is in the domain of the generalization (∀x: Ix ∧ Bx) Rx. But we do not have a resource that actually applies the restricting predicate of this generalization to the term a, so we cannot immediately apply the rule for singular Barbara. To put ourselves in a position to do so, we use attachment at stage 3 to add the required sentence to our inactive resources and then instantiate the premise at stage 4.

In some cases, even the use of attachment rules will not enable us to introduce the auxiliary premises we would need to exploit a restricted universal by the detachment rules. In such cases, we need to resort to a rule for exploiting restricted universals that implements the law for the restricted universal as a premise. It is shown in Figure 7.6.2-4:

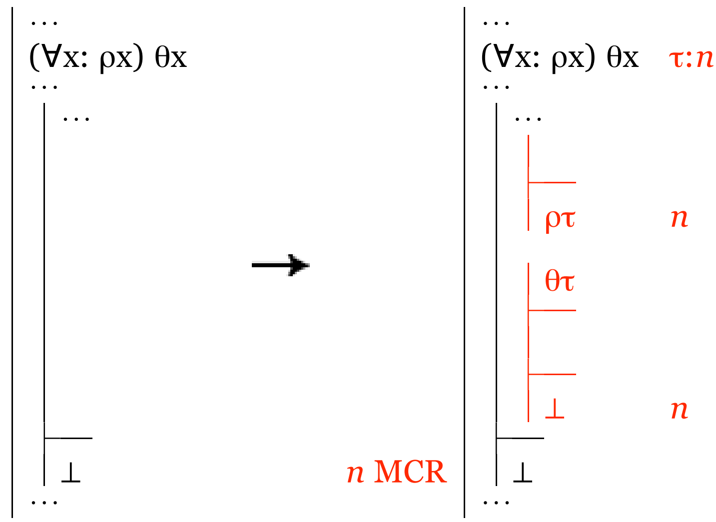

Fig. 7.6.2-4. Developing a *reductio* at stage *n* by exploiting a restricted universal for the term τ.

We can exploit a restricted universal to complete a *reductio* by showing that other resources entail its denial. Here we plan to do this by producing a counterexample, specifically by showing that τ is in the domain of the generalization and reducing to absurdity the claim θτ that it has the attribute. This rule will be named ***Making a Counterexample for Reductio*** (MCR). Of course, the term τ is not the only possible counterexample so the content of the universal is not limited to the claim that τ in particular is not a counterexample. That is why this counts as an exploitation of the universal only for the term τ; our resources may conflict with the universal even if they do not entail that this term is a counterexample to it, so the universal has further potential to contribute to a reduction of our resources to absurdity. A restricted universal (∀x: ρx) θx can be thought of as a general inference ticket that will get us from ρτ to θτ for any term τ. This feature is used in the rule to link the goal of the first gap and the supposition of the second, enabling us to complete the *reductio* if we can close these two gaps. The special virtue of the restricted universal as opposed to a conditional is that it supports an indefinite number of links between gaps rather than merely one.

The comments made about the exploitation rule for unrestricted universals apply to these three exploitation rules as well. First, a universal is never completely exploited though it may be rendered inactive for certain terms. Also, the exploitation of universals should be limited to terms already appearing in the derivation whenever there are such terms. And, indeed, it may be limited to a single term from each alias set.

Another approach to the logical properties of restricted universals simply uses their restatements using unrestricted quantifiers. That is, we take as the basic principle the equivalence

$$(\forall x\text{: } \rho x)\ \theta x \Leftrightarrow \forall x\ (\rho x \rightarrow \theta x)$$

and implement this by two rules, one each for exploiting and planning for a restricted universal simply replace the restricted universal (as an active resource or a goal) by its restatement:

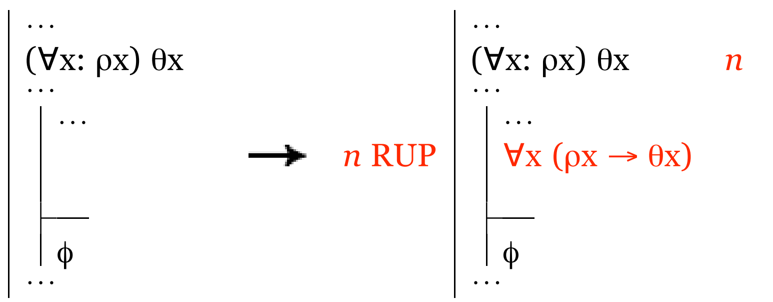

Fig. 7.6.2-5. Developing a derivation at stage *n* by restating a restricted universal resource.

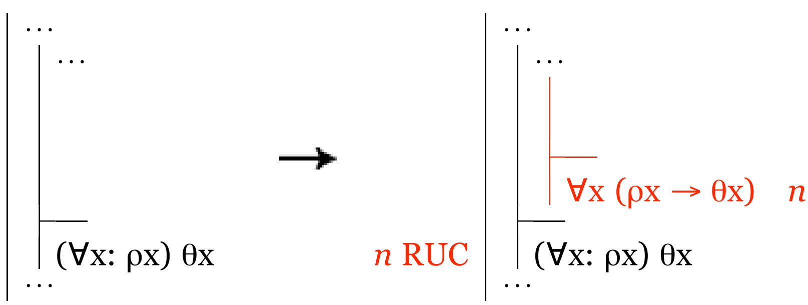

Fig. 7.6.2-6. Developing a derivation at stage *n* by restating a restricted universal goal.

These rules are named ***Restricted Universal Premise*** (RUP) and ***Restricted Universal Conclusion*** (RUC), respectively. Whether they are useful to you will depend on the way you organize your thinking: using them instead of the earlier four rules would mean fewer rules to remember but they will usually need to be combined with two further rules to get the effect of the earlier rules (e.g., RUC with UG and CP to get the effect of RUG). Here is a derivation for the second of the examples above using these rules:

	(∀x: Ix ∧ Bx) Rx	6
	ⓐ	
	Ba	(5)
	Ia	(5)
5 Adj	Ia ∧ Ba	X,(8)
6 RUP	∀x ((Ix ∧ Bx) → Rx)	a:7
7 UI	(Ia ∧ Ba) → Ra	8
8 MPP	Ra	(9)
	●	
9 QED	Ra	4
4 CP	Ia → Ra	3
3 CP	Ba → (Ia → Ra)	2
2 UG	∀x (Bx → (Ix → Rx))	1
1 RUC	(∀x: Bx) (Ix → Rx)	

This has been constructed so that each of the rules for restricted universals is replaced by a series of three steps, a use of rule for the unrestricted quantifier preceded by RUP or RUC and followed by a rule for the conditional. Whether you find this approach to restricted quantifiers easier or harder than the use of the four earlier rules, it is a legitimate one: the rules RUP and RUC are certainly sound and safe since they replace a resource or goal by an equivalent sentence and, while they are not direct since they restate rather than decomposing, they are progressive if we measure distance from the end of a gap in part by the number of restricted universal quantifiers appearing in resources or the goal.

Glen Helman 07 Nov 2004