

7.5.xa. Exercise answers

1.	<i>instance for a</i>	<i>instance for b</i>	<i>instance for c</i>
a. $\forall x Fx$	Fa	Fb	Fc
b. $\forall y Fy$	Fa	Fb	Fc
c. $\forall x Rxa$	Raa	Rba	Rca
d. $\forall x Saxb$	Saab	Sabb	Sacb
e. $\forall x \forall y Rxy$	$\forall y Ray$	$\forall y Rby$	$\forall y Rcy$
f. $\forall x (Fx \rightarrow Gx)$	$Fa \rightarrow Ga$	$Fb \rightarrow Gb$	$Fc \rightarrow Gc$
g. $\forall x (Fx \rightarrow Gd)$	$Fa \rightarrow Gd$	$Fb \rightarrow Gd$	$Fc \rightarrow Gd$
h. $\forall x (Fx \rightarrow \forall y Rxy)$	$Fa \rightarrow \forall y Ray$	$Fb \rightarrow \forall y Rby$	$Fc \rightarrow \forall y Rcy$
i. $\forall x (Fx \rightarrow \forall x Rxx)$	$Fa \rightarrow \forall x Rxx$	$Fb \rightarrow \forall x Rxx$	$Fc \rightarrow \forall x Rxx$

2. a.	$\forall x Fx$	a:1
	$\forall x (Fx \rightarrow Gx)$	a:2
1 UI	Fa	(3)
2 UI	$Fa \rightarrow Ga$	3
3 MPP	Ga	(4)
	•	
4 QED	Ga	
b.	$\forall x (Fx \wedge Gx)$	a:1, b:3
1 UI	$Fa \wedge Ga$	2
2 Ext	Fa	(5)
2 Ext	Ga	
3 UI	$Fb \wedge Gb$	4
4 Ext	Fb	
4 Ext	Gb	(5)
5 Adj	$Fa \wedge Gb$	(6)
	•	
6 QED	$Fa \wedge Gb$	
c.	$\forall x Rxa$	b:1
	$\forall x (Rbx \rightarrow Gx)$	a:2
1 UI	Rba	(3)
2 UI	$Rba \rightarrow Ga$	3
3 MPP	Ga	(4)
	•	
4 QED	Ga	

d.

	$\forall x Fx$	a:2
	$\forall x (Fx \rightarrow Gx)$	a:3
	(a)	
2 UI	Fa	(4)
3 UI	Fa \rightarrow Ga	4
4 MPP	Ga	(5)
	•	
	Ga	1
5 QED		
1 UG	$\forall x Gx$	

e.

	$\forall x (Fx \wedge Gx)$	a:3,b:7		$\forall x Fx \wedge \forall x Gx$	1
	(a)			$\forall x Fx$	a:3
3 UI	Fa \wedge Ga	4	1 Ext	$\forall x Gx$	a:4
4 Ext	Fa		1 Ext	(a)	
4 Ext	Ga	(5)	3 UI	Fa	(5)
	•		4 UI	Ga	(5)
	Fa	2	5 Adj	Fa \wedge Ga	X, (6)
5 QED				•	
2 UG	$\forall x Fx$	1	6 QED	Fa \wedge Ga	2
	(b)		2 UG	$\forall x (Fx \wedge Gx)$	
7 UI	Fb \wedge Gb	8			
8 Ext	Fb				
8 Ext	Gb	(9)			
	•				
	Gb	6			
9 QED					
6 UG	$\forall x Gx$	1			
1 Cnj	$\forall x Fx \wedge \forall x Gx$				

The term a could have been used again as the parameter of the second general argument of the derivation on the left since we require only that a parameter not appear outside its scope line in the gap which is developed by introducing the general argument, and the first general argument is boxed off from the gap that is developed by setting up the second one. But we will not be short of letters to use as parameters, so it will be easier to see that the requirement is satisfied if we use a new parameter for each general argument in a derivation.

f.	$\forall x \forall y Rxy$	$a:1, b:3$
	$\forall y Ray$	$b:2$
1 UI	Rab	(5)
2 UI	$\forall y Rby$	$b:4, a:6$
3 UI	Rbb	(5)
4 UI	$Rab \wedge Rbb$	$X, (7)$
5 Adj	Rba	(7)
6 UI	$(Rab \wedge Rbb) \wedge Rba$	$X, (8)$
7 Adj	•	
	$(Rab \wedge Rbb) \wedge Rba$	
8 QED		

g.	$\forall x \forall y Rxy$	$b:2$
	(b) $\forall y Rby$	$a:3$
2 UI	Rba	(4)
3 UI	•	
	Rba	1
4 QED		
1 UG	$\forall y Rya$	

Notice that the term a cannot be used as the parameter of the general argument in this derivation because it already appears in the gap (specifically, in its goal) when the general argument is introduced.

h.	$\forall x \forall y (Rxy \rightarrow \neg Ryx)$	$a:3$
	(a) Raa	$(5), (6)$
3 UI	$\forall y (Ray \rightarrow \neg Rya)$	$a:4$
4 UI	$Raa \rightarrow \neg Raa$	5
5 MPP	$\neg Raa$	(6)
	•	
6 Nc	\perp	2
	$\neg Raa$	1
2 RAA		
1 UG	$\forall x \neg Rxx$	

i.

	$\forall x \forall y \forall z ((Rxy \wedge Ryz) \rightarrow Rxz)$	a:5
	$\forall x \neg Rxx$	a:10
	(a)	
	(b)	
	Rab	(8)
	Rba	(8)
5 UI	$\forall y \forall z ((Ray \wedge Ryz) \rightarrow Raz)$	b:6
6 UI	$\forall z ((Rab \wedge Rbz) \rightarrow Raz)$	a:7
7 UI	$(Rab \wedge Rba) \rightarrow Raa$	9
8 Adj	$Rab \wedge Rba$	X,(9)
9 MPP	Raa	(11)
10 UI	$\neg Raa$	(11)
	•	
	⊥	4
11 Nc	$\neg Rba$	3
4 RAA	$Rab \rightarrow \neg Rba$	2
3 CP	$\forall y (Ray \rightarrow \neg Rya)$	1
2 UG	$\forall x \forall y (Rxy \rightarrow \neg Ryx)$	
1 UG		