

7.4.xa. Exercise answers

1. a. *Every picture pleased everyone*
Every picture is such that (it pleased everyone)
 $(\forall x: x \text{ is a picture})$ x pleased everyone
 $(\forall x: Cx)$ *everyone is such that (x pleased him or her)*
 $(\forall x: Cx) (\forall y: y \text{ is a person})$ x pleased y

$$(\forall x: Cx) (\forall y: Py) Lxy$$

$$\forall x (Cx \rightarrow \forall y (Py \rightarrow Lxy))$$

[C: $\lambda x (x \text{ is a picture})$; L: $\lambda xy (x \text{ pleased } y)$; P: $\lambda x (x \text{ is a person})$]

- b. *No picture pleased everyone*
No picture is such that (it pleased everyone)
 $(\forall x: x \text{ is a picture}) \neg x$ pleased everyone
 $(\forall x: Cx) \neg$ *everyone is such that (x pleased him or her)*
 $(\forall x: Cx) \neg (\forall y: y \text{ is a person})$ x pleased y

$$(\forall x: Cx) \neg (\forall y: Py) Lxy$$

$$\forall x (Cx \rightarrow \neg \forall y (Py \rightarrow Lxy))$$

[C: $\lambda x (x \text{ is a picture})$; L: $\lambda xy (x \text{ pleased } y)$; P: $\lambda x (x \text{ is a person})$]

- c. *No picture pleased anyone.*
Everyone is such that (no picture pleased him or her)
 $(\forall x: x \text{ is a person})$ no picture pleased x
 $(\forall x: Px)$ *no picture is such that (it pleased x)*
 $(\forall x: Px) (\forall y: y \text{ is a picture}) \neg y$ pleased x

$$(\forall x: Px) (\forall y: Cy) \neg Lyx$$

$$\forall x (Px \rightarrow \forall y (Cy \rightarrow \neg Lyx))$$

[C: $\lambda x (x \text{ is a picture})$; L: $\lambda xy (x \text{ pleased } y)$; P: $\lambda x (x \text{ is a person})$]

Notice that we are forced here to change from *anyone* to *everyone* when using subject-predicate expansion because the result of retaining *anyone* would be awkward at best. In general, although it is not impossible for *anyone* to serve as the subject of a sentence (see **f** below), it is best to avoid using it as the subject of sentence in expanded form.

- d. *Each provision of the law affected every sector of the economy. Each provision of the law is such that (it affected every sector of the economy)*

($\forall x: \underline{x}$ is a provision of the law) x affected every sector of the economy

($\forall x: Px$) every sector of the economy is such that (x affected it)

($\forall x: Px$) ($\forall y: \underline{y}$ is a sector of the economy) x affected y

$$(\forall x: Px) (\forall y: Sye) Axy$$

$$\forall x (Px \rightarrow \forall y (Sye \rightarrow Axy))$$

[A: λxy (x affected y); P: λxy (x is a provision of y); S: λxy (x is a sector of y); e: *the economy*; l: *the law*]

- e. *No picture pleased anyone except photographers. All people except photographers are such that (no picture pleased them)*

[or: *Everyone who is not a photographer is such that (no picture pleased him or her)*]

($\forall x: x$ is a person $\wedge \neg x$ is a photographer) no picture pleased x

($\forall x: Px \wedge \neg Hx$) no picture is such that (it pleased x)

($\forall x: Px \wedge \neg Hx$) ($\forall y: y$ is a picture) $\neg y$ pleased x

$$(\forall x: Px \wedge \neg Hx) (\forall y: Cy) \neg Lyx$$

$$\forall x ((Px \wedge \neg Hx) \rightarrow \forall y (Cy \rightarrow \neg Lyx))$$

[C: λx (x is a picture); H: λx (x is a photographer); L: λxy (x pleased y); P: λx (x is a person)]

The phrase *all people* is used in the first restatement so that it agrees in number with *except photographers*. It has the disadvantage that *no picture pleased them* might be misunderstood to say that no picture pleased them all. (That would be a misunderstanding because *them* used in the context *them all* would need a subject not already containing *all*—something like *people other than photographers*—as its antecedent.) The alternative using *everyone who isn't a photographer* instead of *all people except photographers* is designed to avoid this misunderstanding. In general, it is best to choose a singular subject when using subject-predicate expansion.

- f. *Anyone who likes all mammals likes all horses*
Everyone who likes all mammals is such that (he or she likes all horses)
 $(\forall x: x \text{ is a person} \wedge x \text{ likes all mammals}) \rightarrow x \text{ likes all horses}$
 $(\forall x: Px \wedge \text{every mammal is such that } (x \text{ likes it})) \rightarrow \text{every horse is such that } (x \text{ likes it})$
 $(\forall x: Px \wedge (\forall y: y \text{ is a mammal}) \rightarrow x \text{ likes } y) \rightarrow (\forall z: z \text{ is a horse}) \rightarrow x \text{ likes } z$

$$(\forall x: Px \wedge (\forall y: My) \rightarrow Lxy) \rightarrow (\forall z: Hz) \rightarrow Lxz$$

$$\forall x ((Px \wedge \forall y (My \rightarrow Lxy)) \rightarrow \forall z (Hz \rightarrow Lxz))$$

[H: $\lambda x (x \text{ is a horse})$; L: $\lambda xy (x \text{ likes } y)$; M: $\lambda x (x \text{ is a mammal})$;
P: $\lambda x (x \text{ is a person})$]

- g. *The law stimulated only sectors of the economy that were affected by all the law's provisions*
Only sectors of the economy that were affected by all the law's provisions are such that (the law stimulated them)
 $(\forall x: \neg x \text{ is a sector of the economy that was affected by all the law's provisions}) \rightarrow \text{the law stimulated } \underline{x}$
 $(\forall x: \neg (\underline{x} \text{ is a sector of the economy}) \wedge x \text{ was affected by all the law's provisions}) \rightarrow \text{Tlx}$
 $(\forall x: \neg (Sxe \wedge \text{every provision of the law is such that } (x \text{ was affected by it}))) \rightarrow \text{Tlx}$
 $(\forall x: \neg (Sxe \wedge (\forall y: \underline{y} \text{ is a provision of the law}) \rightarrow x \text{ was affected by } y)) \rightarrow \text{Tlx}$

$$(\forall x: \neg (Sxe \wedge (\forall y: Py) \rightarrow Fxy)) \rightarrow \text{Tlx}$$

$$\forall x (\neg (Sxe \wedge \forall y (Py \rightarrow Fxy)) \rightarrow \neg \text{Tlx})$$

[F: $\lambda xy (x \text{ was affected by } y)$; P: $\lambda xy (x \text{ is a provision of } y)$; S: $\lambda xy (x \text{ is a sector of } y)$; T: $\lambda xy (x \text{ stimulated } y)$; e: *the economy*;
l: *the law*]

- h. *No one who doesn't like all mammals likes any badger.*
Every badger is such that (no one who doesn't like all mammals likes it)
 $(\forall x: x \text{ is badger}) \rightarrow \text{no one who doesn't like all mammals likes } x$
 $(\forall x: Bx) \rightarrow \text{no one who doesn't like all mammals is such that (he or she likes } x)$
 $(\forall x: Bx) \rightarrow (\forall y: y \text{ is a person who doesn't like all mammals}) \rightarrow y \text{ likes } x$
 $(\forall x: Bx) \rightarrow (\forall y: y \text{ is a person} \wedge y \text{ doesn't like all mammals}) \rightarrow Lyx$

$(\forall x: Bx) (\forall y: Py \wedge \neg y \text{ likes all mammals}) \neg Lyx$
 $(\forall x: Bx) (\forall y: Py \wedge \neg \text{every mammal is such that } (y \text{ likes it}))$
 $\neg Lyx$
 $(\forall x: Bx) (\forall y: Py \wedge \neg (\forall z: z \text{ is a mammal}) y \text{ likes } z) \neg Lyx$

$(\forall x: Bx) (\forall y: Py \wedge \neg (\forall z: Mz) Lyz) \neg Lyx$
 $\forall x (Bx \rightarrow \forall y ((Py \wedge \neg \forall z (Mz \rightarrow Lyz)) \rightarrow \neg Lyx))$

[B: $\lambda x (x \text{ is a badger})$; L: $\lambda xy (x \text{ likes } y)$; M: $\lambda x (x \text{ is a mammal})$; P: $\lambda x (x \text{ is a person})$]

i. *Everyone saw everything that anyone saw.*

Everyone is such that (everyone saw everything that he or she saw)

$(\forall x: x \text{ is a person})$ *everyone saw everything that x saw*

$(\forall x: Px)$ *everyone is such that (he or she saw everything that x saw)*

$(\forall x: Px) (\forall y: y \text{ is a person})$ *y saw everything that x saw*

$(\forall x: Px) (\forall y: Py)$ *everything that x saw is such that (y saw it)*

$(\forall x: Px) (\forall y: Py) (\forall z: z \text{ is a thing that x saw})$ *y saw z*

$(\forall x: Px) (\forall y: Py) (\forall z: x \text{ saw } z) Syz$

$(\forall x: Px) (\forall y: Py) (\forall z: Sxz) Syz$
 $\forall x (Px \rightarrow \forall y (Py \rightarrow \forall z (Sxz \rightarrow Syz)))$

[P: $\lambda x (x \text{ is a person})$; S: $\lambda xy (x \text{ saw } y)$]

j. *No one saw anything that anyone liked.*

Everyone is such that (no one saw anything that he or she liked)

$(\forall x: x \text{ is a person})$ *no one saw anything x liked*

$(\forall x: Px)$ *everything x liked is such that (no one saw it)*

$(\forall x: Px) (\forall y: y \text{ is a thing x liked})$ *no one saw y*

$(\forall x: Px) (\forall y: x \text{ liked } y)$ *no one is such that (he or she saw y)*

$(\forall x: Px) (\forall y: Lxy) (\forall z: z \text{ is a person}) \neg z \text{ saw } y$

$(\forall x: Px) (\forall y: Lxy) (\forall z: Pz) \neg Szy$
 $\forall x (Px \rightarrow \forall y (Lxy \rightarrow \forall z (Pz \rightarrow \neg Szy)))$

[L: $\lambda xy (x \text{ liked } y)$; P: $\lambda x (x \text{ is a person})$; S: $\lambda xy (x \text{ saw } y)$]

The quantifier phrases could have been analyzed in a different order to yield an equivalent interpretation but that would have forced us to change one or both of the two *anys* to *some*.

- k. *No one who anyone could recall spoke to everyone.*
Everyone is such that (no one who he or she could recall spoke to everyone)
 ($\forall x$: x is a person) *no one who x could recall spoke to everyone*
 ($\forall x$: Px) *no one who x could recall is such that (he or she spoke to everyone)*
 ($\forall x$: Px) ($\forall y$: y is a person who x could recall) $\neg y$ *spoke to everyone*
 ($\forall x$: Px) ($\forall y$: y is a person $\wedge x$ could recall y) \neg *everyone is such that (y spoke to him or her)*
 ($\forall x$: Px) ($\forall y$: $Py \wedge Rxy$) \neg ($\forall z$: z is a person) y *spoke to z*

$$(\forall x: Px) (\forall y: Py \wedge Rxy) \neg (\forall z: Pz) Syz$$

$$\forall x (Px \rightarrow \forall y ((Py \wedge Rxy) \rightarrow \neg \forall z (Pz \rightarrow Syz)))$$

[P : λx (x is a person); R : λxy (x could recall y); S : λxy (x spoke to y)]

- l. *No one who everyone could recall spoke to anyone*
everyone is such that (no one who everyone could recall spoke to him or her)
 ($\forall x$: x is a person) *no one who everyone could recall spoke to x*
 ($\forall x$: Px) *no one who everyone could recall is such that (he or she spoke to x)*
 ($\forall x$: Px) ($\forall y$: y is a person who everyone could recall) $\neg y$ *spoke to x*
 ($\forall x$: Px) ($\forall y$: y is a person \wedge everyone could recall y) $\neg Syx$
 ($\forall x$: Px) ($\forall y$: $Py \wedge$ everyone is such that (he or she could recall y)) $\neg Syx$
 ($\forall x$: Px) ($\forall y$: $Py \wedge (\forall z$: z is a person) z could recall y) $\neg Syx$

$$(\forall x: Px) (\forall y: Py \wedge (\forall z: Pz) Rzy) \neg Syx$$

$$\forall x (Px \rightarrow \forall y ((Py \wedge \forall z (Pz \rightarrow Rzy)) \rightarrow \neg Syx))$$

[P : λx (x is a person); R : λxy (x could recall y); S : λxy (x spoke to y)]

- m. *Of the pictures anyone saw, no candid ones pleased everyone in them*
Everyone is such that (of the pictures he or she saw, no candid ones pleased everyone in them)
 ($\forall x$: x is a person) *of the pictures x saw, no candid ones pleased everyone in them*
 ($\forall x$: Px) *of the pictures x saw, no candid one is such that (it*

pleased everyone in it)

$(\forall x: Px) (\forall y: y \text{ is a picture } x \text{ saw } \wedge y \text{ is candid}) \rightarrow y \text{ pleased everyone in } y$

$(\forall x: Px) (\forall y: (y \text{ is a picture } \wedge x \text{ saw } y) \wedge y \text{ is candid}) \rightarrow$
everyone in } y \text{ is such that (y pleased him or her)

$(\forall x: Px) (\forall y: (Cy \wedge Sxy) \wedge Dy) \rightarrow (\forall z: z \text{ is a person in } y) y$
pleased } z

$(\forall x: Px) (\forall y: (Cy \wedge Sxy) \wedge Dy) \rightarrow (\forall z: z \text{ is a person } \wedge z \text{ is in } y)$
Lyz

$(\forall x: Px) (\forall y: (Cy \wedge Sxy) \wedge Dy) \rightarrow (\forall z: Pz \wedge Nzy) Lyz$

$\forall x (Px \rightarrow \forall y (((Cy \wedge Sxy) \wedge Dy) \rightarrow \neg \forall z ((Pz \wedge Nzy) \rightarrow Lyz)))$

[C: $\lambda x (x \text{ is a picture})$; D: $\lambda x (x \text{ is candid})$; L: $\lambda xy (x \text{ pleased } y)$;

P: $\lambda x (x \text{ is a person})$; S: $\lambda x (x \text{ saw } y)$]

- n. *No law will affect only sectors of the economy that figure in all its provisions*

No law is such that (it will affect only sectors of the economy that figure in all its provisions)

$(\forall x: x \text{ is a law}) \rightarrow x \text{ will affect only sectors of the economy that figure in all } x\text{'s provisions}$

$(\forall x: Lx) \rightarrow$ *only sectors of the economy that figure in all } x\text{'s provisions are such that (x will affect them)}*

$(\forall x: Lx) \rightarrow (\forall y: \neg y \text{ is a sector of the economy that figures in all } x\text{'s provisions}) \rightarrow x \text{ will affect } y$

$(\forall x: Lx) \rightarrow (\forall y: \neg (y \text{ is a sector of the economy } \wedge y \text{ figures in all } x\text{'s provisions})) \rightarrow Axy$

$(\forall x: Lx) \rightarrow (\forall y: \neg (Sye \wedge \text{all } x\text{'s provisions are such that (y figures in them)})) \rightarrow Axy$

$(\forall x: Lx) \rightarrow (\forall y: \neg (Sye \wedge (\forall z: z \text{ is a provision of } x) y \text{ figures in } z)) \rightarrow Axy$

$(\forall x: Lx) \rightarrow (\forall y: \neg (Sye \wedge (\forall z: Pzx) Fyz)) \rightarrow Axy$

$\forall x (Lx \rightarrow \neg \forall y (\neg (Sye \wedge \forall z (Pzx \rightarrow Fyz)) \rightarrow \neg Axy))$

[A: $\lambda xy (x \text{ affects } y)$; F: $\lambda xy (x \text{ figures in } y)$; L: $\lambda x (x \text{ is a law})$;

P: $\lambda xy (x \text{ is a provision of } y)$; S: $\lambda xy (x \text{ is a sector of } y)$; e: *the economy*]

or (and perhaps better): $(\forall x: Lx) \rightarrow (\forall y: Sye \wedge \neg (\forall z: Pzx) Fyz) \rightarrow Axy$ —this is the result of taking *sectors of the economy* to indicate bounds so that the formula

x will affect only sectors of the economy that figure in all } x\text{'s provisions}

would be expanded to

among sectors of the economy, only those that figure in all x's provisions are such that (x will affect them)

2. a. $\frac{\frac{\frac{\quad}{\forall x Fx} \rightarrow \frac{\frac{\quad}{\forall y Gy}}{\quad}}{\quad}}{\quad}$ b. $\frac{\frac{\frac{\quad}{\forall x (Fx \rightarrow \frac{\frac{\quad}{\forall y Gy}}{\quad})}}{\quad}}{\quad}$
- c. $\frac{\frac{\frac{\frac{\quad}{\forall y (\frac{\frac{\frac{\quad}{\forall x Fx} \rightarrow Gy)}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad}$ d. $\frac{\frac{\frac{\frac{\quad}{\forall y \frac{\frac{\frac{\quad}{\forall x Fx} \rightarrow Gy}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad}$
- e. $\frac{\frac{\frac{\frac{\frac{\quad}{\forall x: \frac{\frac{\frac{\quad}{\forall y Rxy}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad} Fx$ f. $\frac{\frac{\frac{\frac{\frac{\quad}{\forall y (\frac{\frac{\frac{\quad}{\forall x: Rxy}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad} Fx$
- g. $\frac{\frac{\frac{\frac{\frac{\quad}{\forall x: \frac{\frac{\frac{\quad}{Rxy}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad} \forall y Fx$ h. $\frac{\frac{\frac{\frac{\frac{\quad}{\forall x: \frac{\frac{\frac{\quad}{\forall y Rxy}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad}}{\quad} Pxy$

3. a. $(\forall x: x \text{ is a mosquito}) (\forall y: y \text{ is a person}) x \text{ despises } y$
($\forall x: x \text{ is a mosquito}$) every person is such that (x despises him or her)
 $(\forall x: x \text{ is a mosquito}) x \text{ despises every person}$
Every mosquito is such that (it despises every person)

Every mosquito despises every person or: Every mosquito despises all people

- b. $(\forall x: x \text{ is a person}) \neg (\forall y: y \text{ is a mosquito}) x \text{ despises } y$
 $(\forall x: x \text{ is a person}) \neg$ every mosquito is such that (x despises it)
 $(\forall x: x \text{ is a person}) \neg x \text{ despises every mosquito}$
No one is such that (he or she despises every mosquito)

No one despises every mosquito

- c. $(\forall x: x \text{ is a mosquito}) (\forall y: y \text{ is a person}) \neg y \text{ despises } x$
 $(\forall x: x \text{ is a mosquito})$ no person is such that (he or she despises x)
 $(\forall x: x \text{ is a mosquito})$ no one despises x
Every mosquito is such that (no one despises it)

No one despises any mosquito or: No one despises a mosquito

- d. $(\forall x: x \text{ is a person}) (\forall y: y \text{ is a mosquito} \wedge y \text{ has bitten } x) \neg x \text{ despises } y$
 $(\forall x: x \text{ is a person}) (\forall y: y \text{ is a mosquito that has bitten } x) \neg x \text{ despises } y$
 $(\forall x: x \text{ is a person})$ no mosquito that has bitten x is such that (x despises it)

$(\forall x: x \text{ is a person}) x \text{ despises no mosquito that has bitten } x$
Every person is such that (he or she despises no mosquito that has bitten him or her)

A person despises no mosquito that has bitten him or her

The sentence *No one despises any mosquito that has bitten him or her* is equivalent, and more natural, but its closest analysis would take a slightly different form.

- e. $(\forall x: x \text{ is a person} \wedge (\forall y: y \text{ is a mosquito}) x \text{ despises } y) (\forall z: z \text{ is a mosquito}) \neg z \text{ has bitten } x$
($\forall x: x \text{ is a person} \wedge \text{every mosquito is such that } (x \text{ despises it})$) no mosquito is such that (it has bitten x)
($\forall x: x \text{ is a person} \wedge x \text{ despises every mosquito}$) no mosquito has bitten x
($\forall x: x \text{ is a person who despises every mosquito}$) no mosquito has bitten x
Every person who despises every mosquito is such that (no mosquito has bitten him or her)

No mosquito has bitten anyone who despises every mosquito
or: No mosquito has bitten anyone who despises mosquitoes

- f. $\forall x (\forall y: x \text{ is smaller than } y) \neg y \text{ is smaller than } x$
 $\forall x \text{ nothing that } x \text{ is smaller than is such that (it is smaller than } x)$
 $\forall x \text{ nothing that } x \text{ is smaller than is smaller than } x$
Everything is such that (nothing that it is smaller than is smaller than it)

Nothing that anything is smaller than is smaller than it