7.2.3. Compound restrictions

Connectives may appear within generalizations when we analyze their restricting and quantified predicates. What we really analyze in such cases are the bodies of the lambda abstracts to which the quantifiers are applied. The analysis of such formulas and the problems that arise are not much different from those of truth-functional logic though the frequency with which various kinds of problems occur is different.

Since a restricting formula takes the form x *is a* C where C is a common noun together with modifiers, an analysis of it as a truth-functional compound will not be guided initially by English words marking connectives (apart from cases like λx (x *is a boy or girl*) or λx (x *is a non-smoker*) where the noun phrase itself is compounded using them). Indeed, the analysis of restricting formulas will usually be a matter of separating a common noun from its modifiers. As we saw in 2.1.3, considerable care must be taken in separating attributive adjectives from a common noun. The other modifiers we may find with common nouns —prepositional phrases and relative clauses—are less of a problem in this regard. The word *large* in x *is a flea that is large* acquires some of its significance from the word *flea* and should be restated more expansively when we analyze the open sentence to give something like x *is a flea* ∧ x *is large relative to fleas*. Other problems with attributive adjectives are absent or less pressing with relative clauses. While the open sentence x *is a good thief* is ambiguous (referring either to skill as a thief or to some compensating virtue that makes the thief a good person), x *is a thief who is good* probably speaks of compensating virtue and we would tend to use x *is a thief who is good at it* to speak of skill in thievery. The open sentence x *is an alleged murderer*, which does not admit any analysis as a conjunction, does not admit restatement with a relative clause either; x *is a murderer who is alleged to be one* means something different. The latter formula carries the implication x *is a murderer* and may be analyzed as a conjunction.

Once modifiers are separated from the common noun of a class indicator, a whole range of further logical structure may be open to logical analysis. Relative clauses, in particular, can be rich stores of truth-functional structure. For example, *The officer stopped every car that was either speeding or moving slowly and erratically* may be analyzed as follows:

Every car that was either speeding or moving slowly and erratically is such that (the officer stopped it)
(∀x: x *is a car that was either speeding or moving slowly and erratically*) (*the officer stopped* x)
(∀x: x *is a car* ∧ x *was either speeding or moving slowly and erratically*) Tox
(∀x: Cx ∧ (x *was speeding* ∨ x *was moving slowly and erratically*)) Tox
(∀x: Cx ∧ (Sx ∨ (x *was moving slowly* ∧ x *was moving erratically*)))
Tox

(∀x: Cx ∧ (Sx ∨ (Lx ∧ Ex))) Tpx
∀x ((Cx ∧ (Sx ∨ (Lx ∧ Ex))) → Tox)

[C: λx (x *is a car*); E: λx (x *was moving erratically*); L: λx (x *was moving slowly*); S: λx (x *was speeding*); T: λxy (x *stopped* y); o: *the officer*]

There is no special problem in finding the correct truth-functional analysis is this sort of case.

In some cases where we might expect a truth-functional analysis, we do not find one. This happens when a relative clause modifies the dummy class indicator *thing*. We would analyze the open sentence x *is a thing that is red* as we would x *is red*. And, in general, x *is a thing that* ... can be treated as ... x ... where the variable x may appear in any of a number of different positions when we put this into English; x *is a thing that Jack built* amounts to *Jack built* x and x *is a thing Dave sold to Ed* becomes *Dave sold* x *to Ed*. Of course, we can expect *thing* to drop out only when it appears as a dummy restriction (see the discussion of *everything vs. every **thing*** in 7.2.1).

Bounds and exceptions are another source of logical complexity in the restricting formula. To see how to represent them symbolically, let us return to the example that led us to these ideas. The generalization *Among members of the House, all Republicans except Midwesterners supported the bill* is affirmative so its attribute is expressed by its quantified predicate $_1$ *supported the bill* without use of negation; this will serve as the quantified predicate of the symbolic generalization. We found the domain to be the class of members of the House who are Republicans but not Midwesterners. Membership in this domain is expressed by the predicate λx (x *is a House member* ∧ x *is a Republican* ∧ ¬ x *is a Midwesterner*); this is the restricting predicate. Putting the two predicates together, we have the following:

(∀x: x *is a House member* ∧ x *is a Republican* ∧ ¬ x *is a Midwesterner*)
x *supported the bill*

∀x ((x *is a House member* ∧ x *is a Republican* ∧ ¬ x *is a Midwesterner*)
→ x *supported the bill*)

(Parenthetical grouping of the conjuncts is neglected here only to make the result easier to read.)

The general pattern for an direct affirmative generalization with both bounds and exceptions is as follows:

Among B*s, all* C*s except* E*s are such that* ...*they*...

(∀x: x *is a* B ∧ x *is a* C ∧ ¬ x *is an* E) ...x...
∀x ((x *is a* B ∧ x *is a* C ∧ ¬ x *is an* E) → ...x...)

That is, to handle a bounding class picked out by B, we need to conjoin the formula x *is a* B to what we have otherwise. And, to handle a class of exceptions picked out by a term E, we need to conjoin the formula ¬ x *is an* E. The restricting formula of a direct negative generalization would be handled in the same way since the only difference from a corresponding affirmative generalization lies in the quantified formula.

The effect of bounds on complementary generalizations is analogous; the general pattern is this:

Among Bs, only Cs are such that ...they...

(∀x: x *is a* B ∧ ¬ x *is a* C) ¬ ...x...
∀x ((x *is a* B ∧ ¬ *x is a* C) → ¬ ...x...)

While the restricting formula of an unbounded complementary generalization is a negation, here the restricting formula is a *but-not* form.