7.1.4. Kinds of generalizations

All quantifier phrases serve to say something about the extensions of predicates—specifically, about the number of objects in these extensions. But this description applies more naturally to some quantifier phrases than to others. For example, to restate *Every dog likes bones* as a numerical claim, we must resort to *Zero dogs fail to like bones*. We could describe the role of *every dog* in *Every dog likes bones* more naturally by saying that it is used to state a ***generalization*** about the property of liking bones. All the quantifier phrases we will consider in this course could be described, again some more naturally than others, in terms of the making and denying of generalizations. For example, *A dog has been digging in the garden* could, in a pinch, be regarded as the denial of the negative generalization *No dog has been digging in the garden*. We will begin our study of quantifier phrases by looking at them in terms of the idea of generalization. This will make the properties of phrases like *every dog* and *no dog* stand out more clearly than those of phrases like *a dog*. So we will reserve full attention to the latter phrase and its kin for the next chapter, where we return to looking at quantifier phrases as ways of making more obviously numerical claims. For the time being, we will think of sentences like *A dog has been digging in the garden* as statements that claim the existence of falsifying examples, or ***counterexamples***, to the generalizations on which we will focus our attention.

We will begin our study of generalizations by developing some terminology for describing them informally before offering symbolic representations. A generalization claims that a certain property holds of all objects in a certain collection. We will refer to the collection of objects over which the generalization is made as the ***domain*** of the generalization and refer to the property that is said to hold generally as the ***attribute*** of the generalization. The chief problem in analyzing generalizations will be to identify the domain and the attribute. In a simple case, like *Every dog likes bones*, the domain will be the class of objects picked out by the common noun of the quantifier phrase together with its modifiers (here it is the class of dogs), and the attribute will be the property expressed by the predicate to which the quantifier phrase is applied (here it is the property of liking bones). We will refer to the common noun together with its modifiers as the ***class indicator*** of the generalization and the predicate to which the quantifier phrase is applied as the ***quantified predicate***.

In the simple case of *Every dog likes bones* we would have

class indicator
↓
quantifier phrase *Every* *dog* *likes bones* quantified predicate

but with a more complex quantifier phrase we might have something like

Every *large dog in the neighborhood that was outside last night* *was barking*

with modifiers of the common noun included in the class indicator.

There are two common ways that the quantified predicate of a generalization is related to its attribute. When the attribute is the property expressed by the quantified predicate, we will say that the generalization is ***affirmative***. So *Every dog barks* is an affirmative generalization since barking is both the attribute that is said to hold generally of dogs and the property expressed by the predicate λx (x *barks*). On the other hand, *No dog climbs trees* is not affirmative. The domain of this generalization is also the class of dogs but what is said to hold generally of them is that they do not climb trees. That is, the attribute of this generalization is the denial of the property expressed by the quantified predicate. When this is the relation between the quantified predicate and the attribute—i.e., when the attribute is expressed by the negation of the quantified predicate—we will say that the generalization is ***negative***. Notice that this does not characterize the claim made by the generalization so much as the way this claim is expressed. The generalization *Every box was unopened* is affirmative because its attribute is the property of not being opened and this is the property expressed by the quantified predicate λx (x *was unopened*). On the other hand the generalization *No box was opened* is negative even though it makes the same claim.

There are also two common ways the domain of a generalization is related to the class indicator. A ***direct generalization*** is one whose domain is identical with the class of objects picked out by its class indicator; that is, it is identical with the ***indicated class***. Thus, both *Every dog barks* and *No dog climbs trees* are direct because in both cases the domain, dogs, is picked out directly by their class indicators. However, the generalization

Only trucks were advertised

is not direct. Its domain is not the class of trucks, but the class of non-trucks, whose members are said not to have been advertised. To see this, think what the counterexamples to the generalization would be like: non-trucks that were advertised. We will refer to such generalizations as

complementary. A complementary generalization will often have the corresponding direct generalization as an implicature; for example, *Only new listings were distributed* suggests that *all* new listings were distributed. But *Only trucks were advertised* carries no such implicature, and the implicature is easily canceled in other cases; the sentence *Only new listings were distributed and not even all of them were* is not at all incoherent, much less self-contradictory.

We use the term *complementary* for a generalization like *Only trucks were advertised* because its domain is the complement of the indicated class. The ***complement of*** a set X ***relative to*** a set Y is the class of all members of Y that are not in X (see Figure 7.1.4-1). It will sometimes be useful to turn this idea around and think of the complement of X relative to Y as Y ***with*** X ***subtracted***. When only the class X is specified, the complement must be taken relative to the set of all reference values; this yields the ***full complement of*** a set, the class of every value not in the set. Thus a complementary generalization makes a claim about the full complement of the indicated class.

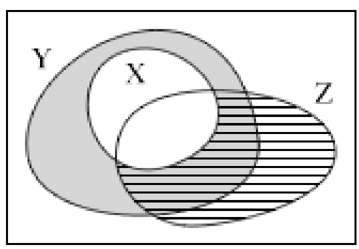

Fig. 7.1.4-1. The complements of a set X relative to sets Y (in gray) and Z (hatched).

The basic classification of generalizations we have considered is summarized in Table 7.1.4-1 below. Each entry shows an English form that can be used as a standard paraphrase for that sort of generalization.

	Affirmative (the attribute is the property expressed by the quantified predicate)	*Negative* (the attribute is the denial of the property expressed by the quantified predicate)
Direct (the domain is the indicated class)	*Every* C *is such that* …*it*…	*No* C *is such that* …*it*…
Complementary (the domain is the complement of the indicated class)		*Only* C*s are such that* …*they*…

Table 7.1.4-1. The classification of generalizations.

There seems to be no quantifier word that indicates an affirmative complementary generalization, but some of the modifying phrases we will consider next can be used to state an affirmative generalization about the complement of a given class.