6.4.1. Extensions and ranges

In this section, we will look at ways of describing the semantic values of the new sorts of expression we have been considering and of using them to present counterexamples to derivations that fail. First, let us collect and sharpen what we know about the semantic values of the several kinds of expression we are considering. Table 6.4.1-1 gives a basic summary that you may compare with the tables of grammatical categories given in 6.1.1 and 6.2.2.

Expression	*Extension*		
sentence	truth value		
term	reference value		
		input	*output*
connective	truth function	truth value(s)	truth value
predicate	attribute	reference value(s)	truth value
functor	reference function	reference value(s)	reference value

Table 6.4.1-1. The extensions of 5 kinds of expression.

In each case the ***intension*** of an expression is a specification of its extension in each possible world. For example, the intension of an individual term is specifies its reference value in each possible world; this is the sort of intensional entity that was mentioned in 6.3.1. In particular, while *George Bush* and *the U. S. president* have the same extension in the actual world, they have different intensions because their extensions differ in other possible worlds.

Since the extensions of the incomplete expressions are functions, they exhibit generality: each such extension determines an output value for each input value from some range of such values. In the case of connectives, the input values are fixed as the two truth values **T** and **F**, and the range of generality of truth functions is thus quite limited. We do not fix the range of reference values, but this range must be known before we know what functions are available as extensions of predicates and functors. We will refer to a specification of the reference values as a ***referential range*** or often simply as a *range*, and we will use the symbol **R** for it. (The word *domain* is often used for this idea, but we will use that word for another concept.) The referential range can be any set that is not empty.

Our logical constants have fixed extensions that we stipulate once and for all. In the case of connectives these are given by their tables. The identity predicate = has an extension that is settled once the referential range is settled: this predicate is true of any pair of reference values whose members are the same but false of any pair of different values. Further basic expressions—unanalyzed sentences, unanalyzed terms other than variables, unanalyzed predicates, and unanalyzed functors—form our ***non-logical vocabulary***, and their extensions are not fixed.

As in truth-functional logic, items of non-logical vocabulary may be assigned extensions by ***extensional interpretations*** or assigned extensions for all possible worlds by ***intensional interpretations***. The extensions assigned to predicates and functors by a given interpretation must have a generality that extends to the same range **R**, so we will speak of an extensional interpretation as being an ***interpretation on*** a range **R**. The basic semantic information needed for the logical forms we are now considering is then a range **R** and an extensional interpretation (on that range) of certain items of non-logical vocabulary; we will refer to this information as a ***structure for*** any expressions that can be formed from this non-logical vocabulary and our logical vocabulary.

It will be convenient to assume that every reference value in the range of a structure comes with a label. We will refer to this label as the ***ID*** of the value. The assumption that all reference values have IDs is actually quite a heavy one. The limitations of decimal notation in capturing irrational numbers like π and the square root of 2 are essential; no system of finite expressions could name all real numbers. So if a range includes all real numbers, its members could not all be labeled by expressions in any ordinary sense. One way around this is to think of IDs as mathematical abstractions—for example, as ID numbers that are not merely numerical expressions but genuine numbers. In this way, the real numbers might be used as their own IDs. But, while these are important theoretical issues that have had considerable impact on the development of logic, they will not affect us practically. Our chief interest will be in structures indicated by the dead end gaps of derivations; and these will all have finite (and usually very small) ranges, so there will be no problem in using numerals (even single-digit numerals) as their IDs.