

## 6.3.s. Summary

The logical properties of identity have two sources, the extension stipulated for  $=$  and the requirement that all predicates and functors be extensional. We will approach these properties by speaking of the terms equated by a true equation as **co-aliases**. Some thought about this idea shows us that identity obeys laws of **reflexivity**, **symmetry**, and **transitivity**. There are other relations obey laws of the same form; any relation that obeys laws of all three sorts is an **equivalence relation** and can be thought to ascribe sameness in certain respects. Identity is distinguished as holding in the fewest cases of any equivalence relation; it implies sameness in respect to all predicates and functors. That is, identity is a ***congruence for*** each **predicate** and **functor**. All of the laws for identity can be summarized by saying that identity is reflexive and is a congruence for all one-place predicates, abstracts included. To say that identity is a congruence for a predicate or functor is to say that it is an extensional operation. A predicate that did not satisfy this requirement would be an **intensional property** (as distinct from a **property in intension**, which is the meaning of an ordinary extensional predicate) and the things of which it was true or false would be **intensional entities**. Whether these ideas are needed to account for aspects of deductive reasoning (or are even coherent) has been a matter of controversy, but we will consider only extensional operations.

A different way of organizing the laws for identity is useful in stating derivation rules. We say that terms are **co-aliases given** a set  $\Delta$  of equations if an equation between the terms follows from  $\Delta$ . A set  $\Delta$  of equations serves to divide a collection of terms into **alias sets**, groups of terms whose members are mutual co-aliases; these are examples of the **equivalence classes** associated with any equivalence relation. The alias sets determined by a given set of equations can be found by a process of **making links between terms**, following rules that implement the laws for identity. As a result, identity obeys a **law for aliases** that says that an equation  $\tau = v$  is entailed by a set of premises if the terms  $\tau$  and  $v$  are co-aliases given the equations among those premises.

The law for aliases and the law of congruence for predicates provide us with the basic derivation rules for  $=$ , each of which is a rule for closing gaps. The rules employ the idea of terms being co-aliases given the equations among the resources of a gap. One, **Equated Co-aliases (EC)**, says a gap may be closed if its goal is an equation between co-aliases, and another, **Distinguished Co-aliases**, says a *reductio* gap may be closed if its resources include a denial of such an equation. A second pair concern predications of the same predicate to series of terms whose

corresponding members are co-aliases. One of these, QED Given Equations (QED=) says that a gap may be closed if its goal is a predication that differs from another predication among the resources only by co-aliases and another, Nc Given Equations (Nc=) says that a *reductio* gap may be closed if one of its resources differs from what another denies only by co-aliases. The statements of these rules use the idea of co-alias series of terms, two series of the same length whose corresponding terms are co-aliases. The idea behind this second pair of rules can be carried further and we may extend any rule by counting as identical, for the purposes of applying the rule, any sentences that differ only by terms that are co-aliases. There are two attachment rules for identity that may be convenient. One, Co-alias Equation (CE), allows us to add to the resources any equation between co-aliases and the other, Congruence (Cng), allows us to add a predication that differs only by co-aliases from one already among the available resources.

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