

5.3.xa. Exercise answers

1. a.

	$B \rightarrow C$	3
	$A \rightarrow B$	2
	A (2)	
2 MPP	B	(3)
3 MPP	C	(4)
	•	
4 QED	C	1
1 PC	$A \rightarrow C$	

b.

	$A \rightarrow B$	2
	$C \leftarrow B$	3
	$C \rightarrow D$	4
	A (2)	
2 MPP	B	(3)
3 MPP	C	(4)
4 MPP	D	(5)
	•	
5 QED	D	1
1 CP	$A \rightarrow D$	

c.

	$A \rightarrow (B \rightarrow C)$	3
	$A \rightarrow B$ 4	
	A	(3),(4)
3 MPP	$B \rightarrow C$	5
4 MPP	B	(5)
5 MPP	C	(6)
	•	
6 QED	C	2
2 CP	$A \rightarrow C$	1
1 CP	$(A \rightarrow B) \rightarrow (A \rightarrow C)$	

d.

	$A \rightarrow (B \rightarrow C)$	3
	$A \rightarrow \neg C$	4
	B	(5)
	A	(3),(4)
3 MPP	$B \rightarrow C$	5
4 MPP	$\neg C$	(6)
5 MPP	C	(6)
•		
6 Nc	\perp	2
2 RAA	$\neg A$	1
1 CP	$B \rightarrow \neg A$	

e.

	$\neg A$	(2)		$A \rightarrow \neg A$	2
	A			A	(2),(3)
	•			$\neg A$	(3)
2 QED	$\neg A$	1	2 MPP	\perp	1
1 CP	$A \rightarrow \neg A$		3 Nc	$\neg A$	
			1 CP	$\neg A$	

f.

	$A \rightarrow B$	2		$\neg B \rightarrow \neg A$	2
	$\neg B$	(2)		A	(2)
2 MTT	$\neg A$	(3)	2 MTT	B	(3)
	•			•	
3 QED	$\neg A$	1	3 QED	B	1
1 CP	$\neg B \rightarrow \neg A$		1 CP	$A \rightarrow B$	

g.

	$A \rightarrow B$	3		$\neg (A \wedge \neg B)$	2
	$A \wedge \neg B$	2		A	(2)
2 Ext	A	(3)	2 MPT	B	(3)
2 Ext	$\neg B$	(4)			
3 MPP	B	(4)		•	
	•			B	1
4 Nc	\perp	1	1 CP	$A \rightarrow B$	
1 RAA	$\neg (A \wedge \neg B)$				

h.

	$A \rightarrow (B \rightarrow C)$	3
	$A \wedge B$	2
2 Ext	A	(3)
2 Ext	B	(4)
3 MPP	$B \rightarrow C$	4
4 MPP	C	(5)
	•	
5 QED	C	1
1 CP	$(A \wedge B) \rightarrow C$	

	$(A \wedge B) \rightarrow C$	4
	A	(5)
	B	(6)
	$\neg C$	(4)
4 MTT	$\neg(A \wedge B)$	5
5 MPT	$\neg B$	(6)
	•	
6 Nc	\perp	3
3 IP	C	2
2 CP	$B \rightarrow C$	1
1 CP	$A \rightarrow (B \rightarrow C)$	

i.

	$(A \rightarrow B) \wedge (A \rightarrow C)$	1
1 Ext	$A \rightarrow B$	3
1 Ext	$A \rightarrow C$	4
	A	(3),(4)
3 MPP	B	(6)
4 MPP	C	(7)
	•	
6 QED	B	5
	•	
7 QED	C	5
5 Cnj	$B \wedge C$	2
2 CP	$A \rightarrow (B \wedge C)$	

	$A \rightarrow (B \wedge C)$	3,7
	A	(3)
3 MPP	$B \wedge C$	4
4 Ext	B	(5)
4 Ext	C	
	•	
5 QED	B	2
2 CP	$A \rightarrow B$	1
	A	
7 MPP	$B \wedge C$	8
8 Ext	B	
8 Ext	C	(9)
	•	
9 QED	C	6
6 QED	$A \rightarrow C$	1
1 Cnj	$(A \rightarrow B) \wedge (A \rightarrow C)$	

- j. Stages 3-5 and 7-11 in the derivation at the right could have taken analogous forms; they are varied here to show two approaches, one using attachment rules and the other without them.

<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border-right: 1px dashed black; padding-right: 5px;"></td> <td style="padding-left: 5px;">$(A \rightarrow C) \wedge (B \rightarrow C)$</td> <td style="padding-left: 5px;">1</td> </tr> <tr> <td style="border-right: 1px dashed black; padding-right: 5px;">1 Ext</td> <td style="padding-left: 5px;">$A \rightarrow C$</td> <td style="padding-left: 5px;">4</td> </tr> <tr> <td style="border-right: 1px dashed black; padding-right: 5px;">1 Ext</td> <td style="padding-left: 5px;">$B \rightarrow C$</td> <td style="padding-left: 5px;">6</td> </tr> <tr> <td style="border-right: 1px dashed black; padding-right: 5px;"></td> <td style="padding-left: 5px;">$A \vee B$</td> <td style="padding-left: 5px;">3</td> </tr> <tr> <td style="border-right: 1px dashed black; padding-right: 5px;"></td> <td style="padding-left: 5px;">A</td> <td style="padding-left: 5px;">(4)</td> </tr> <tr> <td style="border-right: 1px dashed black; padding-right: 5px;">4 MPP</td> <td style="padding-left: 5px;">C</td> <td style="padding-left: 5px;">(5)</td> </tr> <tr> <td style="border-right: 1px dashed black; 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- k. Parallel arguments are again completed differently in the two gaps of each derivation—in the first, to show approaches with attachment rules and without them and, in the second, to show two ways of using attachment rules.

	$(A \rightarrow B) \wedge (B \rightarrow C)$	1		$(A \vee B) \rightarrow (B \wedge C)$	4,10
1 Ext	$A \rightarrow B$	4		A	(3)
1 Ext	$B \rightarrow C$	5,10		$A \vee B$	X,(4)
	$A \vee B$	3	3 Wk	$B \wedge C$	5
	A	(4)	4 MPP	B	(6)
	B	(5)	5 Ext	C	
4 MPP	C	(5)	5 Ext	•	
5 MPP	$B \wedge C$	X,(7)	6 QED	B	2
6 Adj	•		2 CP	$A \rightarrow B$	1
7 QED	$B \wedge C$	3		B	(11)
	B	(9),(10)		$\neg C$	(9)
	•		9 Wk	$\neg (B \wedge C)$	(10)
9 QED	B	8	10 MTT	$\neg (A \vee B)$	(12)
10 MPP	C	(11)	11 Wk	$A \vee B$	(12)
	•			•	
11 QED	C	8	12 Nc	\perp	8
8 Cnj	$B \wedge C$	3	8 IP	C	7
3 PC	$B \wedge C$	2	7 CP	$B \rightarrow C$	1
2 CP	$(A \vee B) \rightarrow (B \wedge C)$		1 Cnj	$(A \rightarrow B) \wedge (B \rightarrow C)$	

2. d. *If Ann was there, then Carol was there if Bill was
Carol wasn't there if Ann was
Ann wasn't there if Bill was*
- f. *If Ann was there, Bill was, too
If Bill wasn't there, Ann wasn't either*
- g. *If Ann was there, Bill was there
Ann wasn't there without Bill being there*
- k. *If Ann was there, Bill was there; and if Bill was there, Carol was
there
If either Ann or Bill was there, then both Bill and Carol were
there*