

5.3.x. Exercise questions

1. Use derivations to establish each of the following. Notice that several are claims of equivalence and require two derivations. All these derivations are designed for the use of detachment rules (especially MPP and MTT), and a number will be quite long if they are not used. Attachment rules from previous chapters will occasionally be useful, and (since we do not yet have a full set of rules for the conditional) they are required in one of the derivations for **k**. Finally, note the leftwards arrow in the second premise of **b**. Although rules like MPP are written using a rightwards arrow they also apply to conditionals written using a leftwards arrow since a conditional $\psi \leftarrow \phi$ is just an alternative way of writing $\phi \rightarrow \psi$ and plays the same role in derivations.
 - a. $B \rightarrow C, A \rightarrow B \Rightarrow A \rightarrow C$
 - b. $A \rightarrow B, C \leftarrow B, C \rightarrow D \Rightarrow A \rightarrow D$
 - c. $A \rightarrow (B \rightarrow C) \Rightarrow (A \rightarrow B) \rightarrow (A \rightarrow C)$
 - d. $A \rightarrow (B \rightarrow C), A \rightarrow \neg C \Rightarrow B \rightarrow \neg A$
 - e. $\neg A \Leftrightarrow A \rightarrow \neg A$
 - f. $A \rightarrow B \Leftrightarrow \neg B \rightarrow \neg A$
 - g. $A \rightarrow B \Leftrightarrow \neg (A \wedge \neg B)$
 - h. $A \rightarrow (B \rightarrow C) \Leftrightarrow (A \wedge B) \rightarrow C$
 - i. $(A \rightarrow B) \wedge (A \rightarrow C) \Leftrightarrow A \rightarrow (B \wedge C)$
 - j. $(A \rightarrow C) \wedge (B \rightarrow C) \Leftrightarrow (A \vee B) \rightarrow C$
 - k. $(A \rightarrow B) \wedge (B \rightarrow C) \Leftrightarrow (A \vee B) \rightarrow (B \wedge C)$
2. Give English sentences illustrating **d**, **f**, **g**, and **k** of 1. (Notice that **k** tells how to restate a particular sort of conjunction of conditionals, one that might be called a *linked conditional*.)