

### 4.3.xa. Exercise answers

**1. a.**

	A ∧ B	1
1 Ext	A	(2)
1 Ext	B	
2 Wk	A ∨ B	X,(3)
	•	
3 QED	A ∨ B	

**b.**

	A ∧ B	1
1 Ext	A	
1 Ext	B	(2)
2 Wk	B ∨ C	X,(3)
	•	
3 QED	B ∨ C	

**c.**

	A ∨ B	1
	¬ A	(1)
1 MTP	B	(2)
	•	
2 QED	B	

**d.** Although the following is a possible approach, the derivation in 4.2.xa is probably more natural:

	A ∨ (A ∧ B)	2
	¬ A	(2),(4)
2 MTP	A ∧ B	3
3 Ext	A	(4)
3 Ext	B	
	•	
4 Nc	⊥	1
1 IP	A	

**e.**

	A ∨ B	3
	¬ (A ∧ C)	2
	¬ (B ∧ C)	4
	C	(2),(5)
2 MPT	¬ A	(3)
3 MTP	B	(4)
4 MPT	¬ C	(5)
	•	
5 Nc	⊥	1
1 RAA	¬ C	

f.

	$A \wedge (B \vee C)$	1
1 Ext	A	(4)
1 Ext	$B \vee C$	3
	$\neg C$	(3)
3 MTP	B	(4)
4 Adj	$A \wedge B$	X,(5)
	•	
5 QED	$A \wedge B$	2
2 PE	$(A \wedge B) \vee C$	

OR

	$A \wedge (B \vee C)$	1
1 Ext	A	(3)
1 Ext	$B \vee C$	2
	B	(3)
3 Adj	$A \wedge B$	X,(4)
4 Wk	$(A \wedge B) \vee C$	X,(5)
	•	
5 QED	$(A \wedge B) \vee C$	2
	C	(6)
6 Wk	$(A \wedge B) \vee C$	X,(7)
	•	
7 QED	$(A \wedge B) \vee C$	2
2 PC	$(A \wedge B) \vee C$	

g.

	$A \vee B$	1
	C	(2),(5)
	A	(2)
2 Adj	$A \wedge C$	X,(3)
3 Wk	$(A \wedge C) \vee (B \wedge C)$	X,(4)
	•	
4 QED	$(A \wedge C) \vee (B \wedge C)$	1
	B	(5)
5 Adj	$B \wedge C$	X,(6)
6 Wk	$(A \wedge C) \vee (B \wedge C)$	X,(7)
	•	
7 QED	$(A \wedge C) \vee (B \wedge C)$	1
1 PC	$(A \wedge C) \vee (B \wedge C)$	

OR

	$A \vee B$	1
	C	(2),(4)
	$\neg (A \wedge C)$	2
2 MPT	$\neg A$	(3)
3 MTP	B	(4)
4 Adj	$B \wedge C$	X,(5)
	•	
5 QED	$B \wedge C$	1
1 PE	$(A \wedge C) \vee (B \wedge C)$	

h.

	$A \vee B$	1
	$\neg A \vee C$	2
	A	(2)
2 MTP	C	(3)
3 Wk	$B \vee C$	X,(4)
	•	
4 QED	$B \vee C$	1
	B	(5)
5 Wk	$B \vee C$	X,(6)
	•	
6 QED	$B \vee C$	1
1 PC	$B \vee C$	

OR

	$A \vee B$	2
	$\neg A \vee C$	3
	$\neg B$	(2)
2 MTP	A	(3)
3 MTP	C	(4)
	•	
4 QED	C	1
1 PE	$B \vee C$	

<b>i.</b>	<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-bottom: 1px solid black; padding: 2px;">A</td><td style="padding-left: 10px;">(2),(3)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>\neg(A \wedge B)</math></td><td style="padding-left: 10px;">2</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>\neg B</math></td><td style="padding-left: 10px;">(3)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>A \wedge \neg B</math></td><td style="padding-left: 10px;">X,(4)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  •</td><td></td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>A \wedge \neg B</math></td><td style="padding-left: 10px;">1</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>(A \wedge B) \vee (A \wedge \neg B)</math></td><td></td></tr> </table>	A	(2),(3)	$\neg(A \wedge B)$	2	$\neg B$	(3)	$A \wedge \neg B$	X,(4)	•		$A \wedge \neg B$	1	$(A \wedge B) \vee (A \wedge \neg B)$		<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-bottom: 1px solid black; padding: 2px;"><math>(A \wedge B) \vee (A \wedge \neg B)</math></td><td style="padding-left: 10px;">3</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>\neg A</math></td><td style="padding-left: 10px;">(2),(5)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>\neg(A \wedge B)</math></td><td style="padding-left: 10px;">X,(3)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>A \wedge \neg B</math></td><td style="padding-left: 10px;">4</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  A</td><td style="padding-left: 10px;">(5)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>\neg B</math></td><td></td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  •</td><td></td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>\perp</math></td><td style="padding-left: 10px;">3</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  A</td><td></td></tr> </table>	$(A \wedge B) \vee (A \wedge \neg B)$	3	$\neg A$	(2),(5)	$\neg(A \wedge B)$	X,(3)	$A \wedge \neg B$	4	A	(5)	$\neg B$		•		$\perp$	3	A	
A	(2),(3)																																	
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Although the derivation above for the second entailment is possible, the derivation for it in 4.2.xa is probably more natural

<b>2. a.</b>	<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-bottom: 1px solid black; padding: 2px;">A <math>\vee</math> A</td><td style="padding-left: 10px;">2</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>\neg A</math></td><td style="padding-left: 10px;">(2),(3)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  A</td><td style="padding-left: 10px;">(3)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  •</td><td></td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>\perp</math></td><td style="padding-left: 10px;">1</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  A</td><td></td></tr> </table>	A $\vee$ A	2	$\neg A$	(2),(3)	A	(3)	•		$\perp$	1	A		<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-bottom: 1px solid black; padding: 2px;">A</td><td style="padding-left: 10px;">(1)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>A \vee A</math></td><td style="padding-left: 10px;">X,(2)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  •</td><td></td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>A \vee A</math></td><td></td></tr> </table>	A	(1)	$A \vee A$	X,(2)	•		$A \vee A$	
A $\vee$ A	2																					
$\neg A$	(2),(3)																					
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2 MTP																						
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Another somewhat artificial approach.

<b>b.</b>	<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-bottom: 1px solid black; padding: 2px;">A <math>\vee</math> B</td><td style="padding-left: 10px;">1</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  A</td><td style="padding-left: 10px;">(2)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  B <math>\vee</math> A</td><td style="padding-left: 10px;">X,(3)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  •</td><td></td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  B <math>\vee</math> A</td><td style="padding-left: 10px;">1</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  B</td><td style="padding-left: 10px;">(4)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  B <math>\vee</math> A</td><td style="padding-left: 10px;">X,(5)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  •</td><td></td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  B <math>\vee</math> A</td><td style="padding-left: 10px;">1</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  B <math>\vee</math> A</td><td></td></tr> </table>	A $\vee$ B	1	A	(2)	B $\vee$ A	X,(3)	•		B $\vee$ A	1	B	(4)	B $\vee$ A	X,(5)	•		B $\vee$ A	1	B $\vee$ A		<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-bottom: 1px solid black; padding: 2px;">B <math>\vee</math> A</td><td style="padding-left: 10px;">2</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  <math>\neg A</math></td><td style="padding-left: 10px;">(2)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  B</td><td style="padding-left: 10px;">(3)</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  •</td><td></td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  B</td><td style="padding-left: 10px;">1</td></tr> <tr><td style="border-bottom: 1px solid black; padding: 2px;">  A <math>\vee</math> B</td><td></td></tr> </table>	B $\vee$ A	2	$\neg A$	(2)	B	(3)	•		B	1	A $\vee$ B	
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As was the case with the derivations in 4.2.xa, each of the above approaches could have been used for both entailments.

c.

	$(A \vee B) \vee C$	3
	$\neg A$	(4)
	$\neg C$	(3)
3 MTP	$A \vee B$	4
4 MTP	$B$	(5)
	•	
5 QED	$B$	2
2 PE	$B \vee C$	1
1 PE	$A \vee (B \vee C)$	

The derivation at the right can be compared to the one in [4.2.3](#)

	$A \vee (B \vee C)$	1
	$A$	(2)
2 Wk	$A \vee B$	X,(3)
3 Wk	$(A \vee B) \vee C$	X,(4)
	•	
4 QED	$(A \vee B) \vee C$	1
	$B \vee C$	5
	$B$	(6)
6 Wk	$A \vee B$	X,(7)
7 Wk	$(A \vee B) \vee C$	X,(8)
	•	
8 QED	$(A \vee B) \vee C$	5
	$C$	(9)
9 Wk	$(A \vee B) \vee C$	(10)
	•	
10 QED	$(A \vee B) \vee C$	5
5 PC	$(A \vee B) \vee C$	1
1 PC	$(A \vee B) \vee C$	

d.

	$A \vee (B \wedge \neg B)$	2		$A$	(1)
	$\neg A$	(2)	1 Wk	$A \vee (B \wedge \neg B)$	X,(2)
2 MTP	$B \wedge \neg B$	3		•	
3 Ext	$B$	(4)	2 QED	$A \vee (B \wedge \neg B)$	
3 Ext	$\neg B$	(4)			
	•				
4 Nc	$\perp$	4			
1 IP	$A$				

e.

	$\neg (A \vee B)$	(4),(7)		$\neg A \wedge \neg B$	1
	$A$	(3)	1 Ext	$\neg A$	(3)
3 Wk	$A \vee B$	X,(4)	1 Ext	$\neg B$	(4)
	•			$A \vee B$	3
4 Nc	$\perp$	2	3 MTP	$B$	(4)
2 RAA	$\neg A$	1		•	
	$B$	(6)	4 Nc	$\perp$	2
6 Wk	$A \vee B$	X,(7)	2 RAA	$\neg (A \vee B)$	
	•				
7 Nc	$\perp$	5			
5 RAA	$\neg B$	1			
1 Cnj	$\neg A \wedge \neg B$				

f.

<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;"><math>\neg(A \wedge B)</math></td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  A</td><td style="padding: 5px;">(2)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>\neg B</math></td><td style="padding: 5px;">(3)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  •</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>\neg B</math></td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>\neg A \vee \neg B</math></td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">1 PE</td><td></td></tr> </table>	$\neg(A \wedge B)$	2	A	(2)	$\neg B$	(3)	•		$\neg B$	1	$\neg A \vee \neg B$		1 PE		<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;"><math>\neg A \vee \neg B</math></td><td style="padding: 5px;">3</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  A <math>\wedge</math> B</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  A</td><td style="padding: 5px;">(3)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  B</td><td style="padding: 5px;">(4)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>\neg B</math></td><td style="padding: 5px;">(4)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  •</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>\perp</math></td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">4 Nc</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">1 RAA</td><td style="padding: 5px;"><math>\neg(A \wedge B)</math></td></tr> </table>	$\neg A \vee \neg B$	3	A $\wedge$ B	2	A	(3)	B	(4)	$\neg B$	(4)	•		$\perp$	1	4 Nc		1 RAA	$\neg(A \wedge B)$
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3. a. This derivation is unchanged from 4.2.xa

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T T	Ⓣ	Ⓣ																																			
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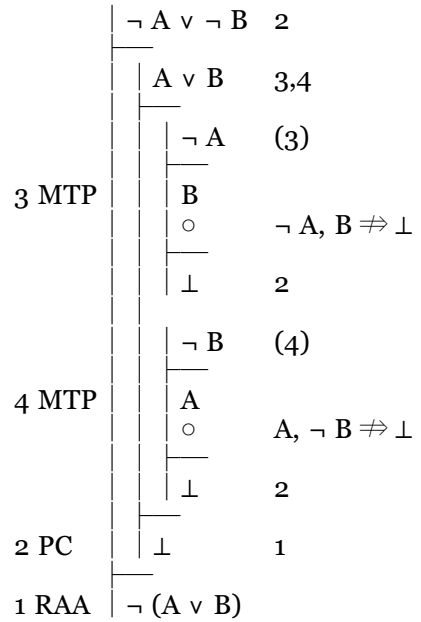
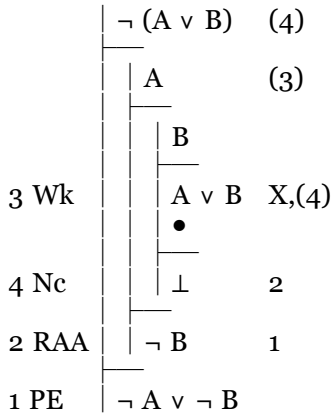
b.

<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;"><math>A \vee (B \wedge C)</math></td><td style="padding: 5px;">3,8</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>\neg A</math></td><td style="padding: 5px;">(3)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  B <math>\wedge</math> C</td><td style="padding: 5px;">4</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  B</td><td style="padding: 5px;">(5)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  C</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  •</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  B</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>A \vee B</math></td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>\neg C</math></td><td style="padding: 5px;">(7)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>\neg(B \wedge C)</math></td><td style="padding: 5px;">X,(8)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  A</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  ○</td><td style="padding: 5px;"><math>A, \neg C \not\Rightarrow \perp</math></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>\perp</math></td><td style="padding: 5px;">9</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  C</td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">1 Cnj</td><td style="padding: 5px;"><math>(A \vee B) \wedge C</math></td></tr> </table>	$A \vee (B \wedge C)$	3,8	$\neg A$	(3)	B $\wedge$ C	4	B	(5)	C		•		B	2	$A \vee B$	1	$\neg C$	(7)	$\neg(B \wedge C)$	X,(8)	A		○	$A, \neg C \not\Rightarrow \perp$	$\perp$	9	C	1	1 Cnj	$(A \vee B) \wedge C$	<table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;"><math>(A \vee B) \wedge C</math></td><td style="padding: 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  A <math>\vee</math> B</td><td style="padding: 5px;">3</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  C</td><td style="padding: 5px;">(4)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>\neg A</math></td><td style="padding: 5px;">(3)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  B</td><td style="padding: 5px;">(4)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  B <math>\wedge</math> C</td><td style="padding: 5px;">X,(5)</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  •</td><td></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  B <math>\wedge</math> C</td><td style="padding: 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">  <math>A \vee (B \wedge C)</math></td><td></td></tr> </table> <p>Each of the following divides the one open gap:</p> <table style="border-collapse: collapse; width: 100%;"> <tr><td style="border-right: 1px solid black; padding: 5px;">A B C</td><td style="border-right: 1px solid black; padding: 5px;"><math>A \vee (B \wedge C) / (A \vee B) \wedge C</math></td><td style="padding: 5px;"></td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">T T F</td><td style="border-right: 1px solid black; padding: 5px;">Ⓣ F</td><td style="padding: 5px;">T Ⓣ</td></tr> <tr><td style="border-right: 1px solid black; padding: 5px;">T F F</td><td style="border-right: 1px solid black; padding: 5px;">Ⓣ F</td><td style="padding: 5px;">T Ⓣ</td></tr> </table>	$(A \vee B) \wedge C$	1	A $\vee$ B	3	C	(4)	$\neg A$	(3)	B	(4)	B $\wedge$ C	X,(5)	•		B $\wedge$ C	2	$A \vee (B \wedge C)$		A B C	$A \vee (B \wedge C) / (A \vee B) \wedge C$		T T F	Ⓣ F	T Ⓣ	T F F	Ⓣ F	T Ⓣ
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Although the use of Wk and MTP shortens the whole first derivation, it

actually delays the dead end, which would have been reached after stage 7 if the first premise had been exploited by PC in the second gap. As in 4.2.xa, the second derivation is unnecessary once a dead-end gap is found in the first.

c.



The following divide the first and second open gap, respectively:

A B	$\neg A \vee \neg B$	$\neg(A \vee B)$		
F T	T	⊖ F	⊖	T
T F	F	⊖ T	⊖	T