

4.3.s. Summary

While a disjunction does not settle the truth values of its disjuncts, it says enough about them that adding the information that one is false will tell us that the other is true. This principle is known traditionally as *modus tollendo ponens*. Since each disjunct entails the disjunction, we know that, if one disjunct is false, then the disjunction and the other disjunct provide the same information. This idea is implemented in a further rule for exploiting disjunctions, also known as *Modus Tollendo Ponens* (MTP). The *not-both* form $\neg(\phi \wedge \psi)$ is analogous to disjunction and analogous principles apply. Specifically, a principle *modus ponendo tollens* tells us that $\neg(\phi \wedge \psi)$ together with the assertion of one of ϕ and ψ entails the denial of the other. And, since the denial of either ϕ or ψ entails $\neg(\phi \wedge \psi)$, we can have a rule *Modus Ponendo Tollens* (MPT) for exploiting *not-both* forms. The rules MTP and MPT are examples of *detachment rules*. The resource exploited in each is its *main resource* and the additional resource that must be available is the *auxiliary resource*.

We will refer to as *weakening* the principle that disjunctions and *not-both* forms are entailed by assertions of components (in the case of disjunctions) or their denials (in the case of the *not-both* form). This principle provides the basis for two further attachment rules, both called *Weakening* (Wk), that license the addition of inactive resources. Since the second resource we must have in order to apply a detachment rule need only be available, attachment rules can be used to prepare for the use of detachment rules as well to prepare for the use of rules that close gaps.

We now have examples of all the types of rules we will employ:

Rules for developing gaps		
	for resources	for goals
atomic sentence		IP
negation $\neg \phi$	CR (if ϕ is not atomic and the goal is \perp)	RAA
conjunction $\phi \wedge \psi$	Ext	Cnj
disjunction $\phi \vee \psi$	PC	PE

Detachment rules (optional)		
main resource	auxiliary resource	rule
$\phi \vee \psi$	$\bar{\phi}$ or $\bar{\psi}$	MTP
$\neg (\phi \wedge \psi)$	ϕ or ψ	MPT

Rules for closing gaps	
when to close	rule
the goal is also a resource	QED
sentences ϕ and $\neg \phi$ are resources & the goal is \perp	Nc
\top is the goal	ENV
\perp is a resource	EFQ

Attachment rules	
added resource	rule
$\phi \wedge \psi$	Adj
$\phi \vee \psi$	Wk
$\neg (\phi \wedge \psi)$	Wk
Rule for lemmas	
prerequisite	rule
the goal is \perp	LFR

Basic system

Added rules (optional)