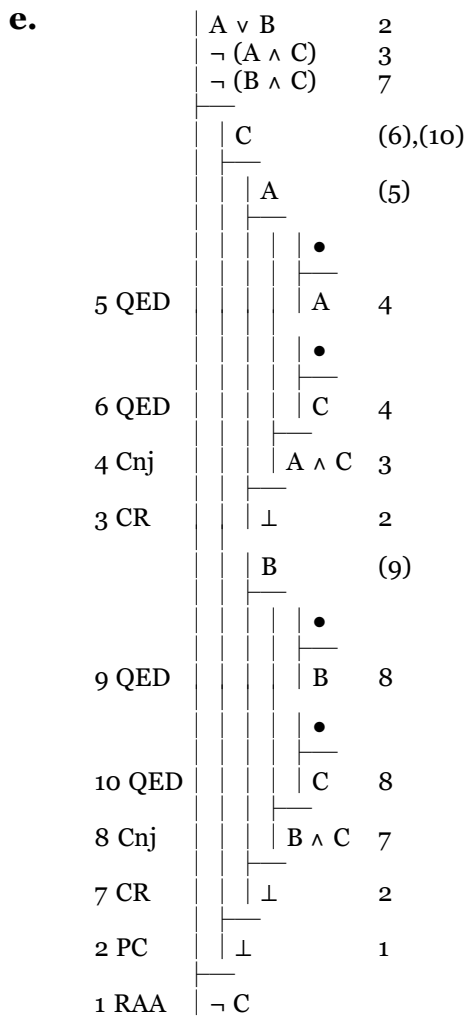
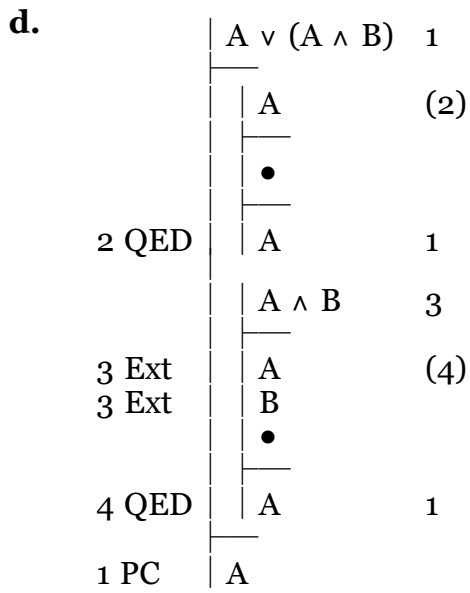


4.2.xa. Exercise answers

1. a.	$A \wedge B$	1
1 Ext	A	
1 Ext	B	(3)
	$\neg A$	
	•	
3 QED	B	2
2 PE	$A \vee B$	

b.	$A \wedge B$	1
1 Ext	A	
1 Ext	B	(3)
	$\neg C$	
	•	
3 QED	B	2
2 PE	$B \vee C$	

c.	$A \vee B$	1
	$\neg A$	(3)
	A	(3)
	$\neg B$	
	•	
3 Nc	\perp	2
2 IP	B	1
	B	(4)
	•	
4 QED	B	1
1 PC	B	



f.

	$A \wedge (B \vee C)$	1
1 Ext	A	(5)
1 Ext	$B \vee C$	2
	B	(6)
	$\neg C$	
	•	
5 QED	A	4
	•	
6 QED	B	4
4 Cnj	$A \wedge B$	3
3 PE	$(A \wedge B) \vee C$	2
	C	(8)
	$\neg (A \wedge B)$	
	•	
8 QED	C	7
7 PE	$(A \wedge B) \vee C$	2
2 PC	$(A \wedge B) \vee C$	

g.

	$A \vee B$	1
	C	(5),(9)
	A	(4)
	$\neg (B \wedge C)$	
	•	
4 QED	A	3
	•	
5 QED	C	3
3 Cnj	$A \wedge C$	2
2 PE	$(A \wedge C) \vee (B \wedge C)$	1
	B	(8)
	$\neg (A \wedge C)$	
	•	
8 QED	B	7
	•	
9 QED	C	7
7 Cnj	$B \wedge C$	6
6 PE	$(A \wedge C) \vee (B \wedge C)$	1
1 PC	$(A \wedge C) \vee (B \wedge C)$	

h.

	$A \vee B$	1
	$\neg A \vee C$	2
	A	(5)
	<div style="border-left: 1px solid black; padding-left: 5px;">$\neg A$</div>	(5)
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">$\neg B$</div>	
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;"> <div style="border-left: 1px solid black; padding-left: 5px;">$\neg C$</div> </div>	
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;"> <div style="border-left: 1px solid black; padding-left: 5px;">•</div> </div>	
5 Nc	\perp	4
4 IP	C	3
3 PE	$B \vee C$	2
	C	(7)
	<div style="border-left: 1px solid black; padding-left: 5px;">$\neg B$</div>	
	<div style="border-left: 1px solid black; padding-left: 5px;">•</div>	
7 QED	C	6
6 PE	$B \vee C$	2
2 PC	$B \vee C$	1
	B	(9)
	<div style="border-left: 1px solid black; padding-left: 5px;">$\neg C$</div>	
	<div style="border-left: 1px solid black; padding-left: 5px;">•</div>	
9 QED	B	8
8 PE	$B \vee C$	1
1 PC	$B \vee C$	

i.

	A	(3),(7)		$(A \wedge B) \vee (A \wedge \neg B)$	1
	$\neg (A \wedge B)$	5		$A \wedge B$	2
	<div style="border-left: 1px solid black; padding-left: 5px;">•</div>		2 Ext	B	(3)
3 QED	A	2	2 Ext	•	
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">B</div>	(8)		A	1
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;"> <div style="border-left: 1px solid black; padding-left: 5px;">•</div> </div>		3 QED	$A \wedge \neg B$	4
7 QED	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;">A</div>	6	4 Ext	A	(5)
	<div style="border-left: 1px solid black; padding-left: 5px; border-bottom: 1px solid black;"> <div style="border-left: 1px solid black; padding-left: 5px;">•</div> </div>		4 Ext	$\neg B$	
8 QED	B	6		•	
6 Cnj	$A \wedge B$	5	5 QED	A	1
5 CR	\perp	4	1 PC	A	
4 RAA	$\neg B$	2			
2 Cnj	$A \wedge \neg B$	1			
1 PE	$(A \wedge B) \vee (A \wedge \neg B)$				

c.

	$(A \vee B) \vee C$	3		
	$\neg A$	(6)		
	$\neg B$	(8)		
	$A \vee B$	4		
	A	(6)		
	$\neg C$			
	•			
6 Nc	\perp	5		
5 IP	C	4		
	B	(8)		
	$\neg C$			
	•			
8 Nc	\perp	7		
7 IP	C	4		
4 PC	C	3		
	C	(9)		
	•			
9 QED	C	3		
3 PC	C	2		
2 PE	$B \vee C$	1		
1 PE	$A \vee (B \vee C)$			

This is the second of the two derivations needed; the first appears in [4.2.3](#). In that one, disjunctive resources are exploited before disjunctive goals are planned for while the derivation at the left here illustrates the opposite approach.

d.

	$A \vee (B \wedge \neg B)$	1		
	A	(2)		
	•			
2 QED	A	1	2 QED	$\neg (B \wedge \neg B)$
	$B \wedge \neg B$	3		•
3 Ext	B	(5)	1 PE	$A \vee (B \wedge \neg B)$
3 Ext	$\neg B$	(5)		A
	$\neg A$			•
	•			\perp
5 Nc	\perp	4		A
4 IP	A	1		A
1 PC	A			A

e.

	$\neg(A \vee B)$	3,7
	A	(5)
	$\neg B$	
	•	
5 QED	A	4
4 PE	$A \vee B$	3
3 CR	\perp	2
2 RAA	$\neg A$	1
	B	(9)
	$\neg A$	
	•	
9 QED	B	8
8 PE	$A \vee B$	7
7 CR	\perp	6
6 RAA	$\neg B$	1
1 Cnj	$\neg A \wedge \neg B$	

	$\neg A \wedge \neg B$	1
1 Ext	$\neg A$	(4)
1 Ext	$\neg B$	(5)
	$A \vee B$	3
	A	(4)
	•	
4 Nc	\perp	3
	B	(5)
	•	
	\perp	3
3 PC	\perp	2
2 RAA	$\neg(A \vee B)$	

f.

	$\neg(A \wedge B)$	3
	A	(5)
	B	(6)
	•	
5 QED	A	4
	•	
6 QED	B	4
4 Cnj	$A \wedge B$	3
3 CR	\perp	2
2 RAA	$\neg B$	1
1 PE	$\neg A \vee \neg B$	

	$\neg A \vee \neg B$	3
	$A \wedge B$	2
2 Ext	A	(4)
2 Ext	B	(5)
	$\neg A$	(4)
	•	
4 Nc	\perp	3
	$\neg B$	(5)
	•	
5 Nc	\perp	3
3 PC	\perp	1
1 RAA	$\neg(A \wedge B)$	

3. a.

$A \vee B$	2	$A \ B \mid A \vee B, A / \neg B$	$\text{T} \ \text{T}$	T	T	F
A			$\text{T} \ \text{T}$	T	T	F

B		A	\circ	$A, B \neq \perp$
\perp	2	B	\circ	$A, B \neq \perp$
\perp	2	\perp		1

2 PC

1 RAA $\neg B$

b.

$A \vee (B \wedge C)$	3,8	$\neg A$	(5)
\perp	4	A	(5)
B	3	$\neg B$	
$B \wedge C$	7	\bullet	
B	3	\perp	4
B	2	B	3
$A \vee B$	1	$B \wedge C$	7
A		B	3
$\neg C$		\bullet	
\perp	9	\perp	$A, \neg C \neq \perp$
C	8	\perp	9
$B \wedge C$	10	C	8
B	11	$B \wedge C$	10
C	8	\bullet	11
C	1	C	8
$(A \vee B) \wedge C$		\perp	9

5 Nc

4 IP

6 Ext

6 Ext

7 QED

3 PC

2 PE

9 IP

10 Ext

10 Ext

11 QED

8 PC

1 Cnj

Since entailment fails in one direction, equivalence must fail, so a second derivation for entailment in the other direction need not be pursued; but that entailment does hold, as is shown below.

$(A \vee B) \wedge C$	1
$A \vee B$	2
C	(8)
A	(4)
$\neg (B \wedge C)$	
\bullet	
A	3
$A \vee (B \wedge C)$	2
B	(7)
$\neg A$	
\bullet	
B	6
\bullet	
C	6
$B \wedge C$	5
$A \vee (B \wedge C)$	2
$A \vee (B \wedge C)$	2

1 Ext

1 Ext

4 QED

3 PE

7 QED

8 QED

6 Cnj

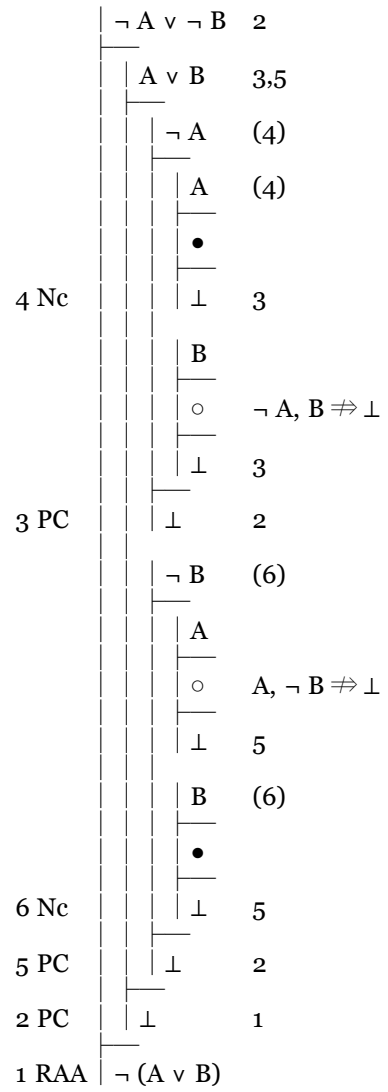
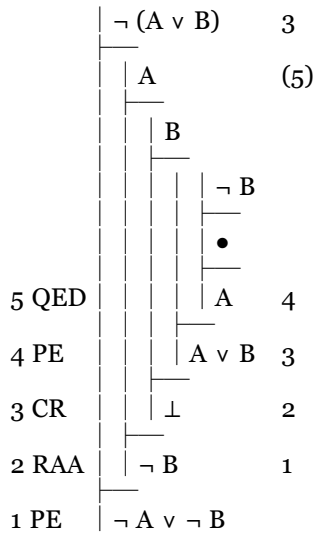
5 PE

2 PC

Each of the following divides the one open gap:

$A \ B \ C$	$A \vee (B \wedge C) / (A \vee B) \wedge C$
$\text{T} \ \text{T} \ \text{F}$	$\text{T} \ \text{F} \ \text{T} \ \text{F}$
$\text{T} \ \text{F} \ \text{F}$	$\text{T} \ \text{F} \ \text{T} \ \text{F}$

c.



The following divide the first and second open gap, respectively:

A	B	$\neg A \vee \neg B$	$\neg(A \vee B)$
F	T	Ⓣ	Ⓣ
T	F	Ⓣ	Ⓣ
T	T	Ⓣ	Ⓣ