

4.2.x. Exercises

1. Use derivations to establish each of the claims of entailment and equivalence shown below. (Remember that claims of equivalence require derivations in both directions.)
 - a. $A \wedge B \Rightarrow A \vee B$
 - b. $A \wedge B \Rightarrow B \vee C$
 - c. $A \vee B, \neg A \Rightarrow B$
 - d. $A \vee (A \wedge B) \Rightarrow A$
 - e. $A \vee B, \neg (A \wedge C), \neg (B \wedge C) \Rightarrow \neg C$
 - f. $A \wedge (B \vee C) \Rightarrow (A \wedge B) \vee C$
 - g. $A \vee B, C \Rightarrow (A \wedge C) \vee (B \wedge C)$
 - h. $A \vee B, \neg A \vee C \Rightarrow B \vee C$
 - i. $A \Leftrightarrow (A \wedge B) \vee (A \wedge \neg B)$
2. Use derivations to establish each of the claims of equivalence below.
 - a. $A \vee A \Leftrightarrow A$
 - b. $A \vee B \Leftrightarrow B \vee A$
 - c. $A \vee (B \vee C) \Leftrightarrow (A \vee B) \vee C$
 - d. $A \vee (B \wedge \neg B) \Leftrightarrow A$
 - e. $\neg (A \vee B) \Leftrightarrow \neg A \wedge \neg B$
 - f. $\neg (A \wedge B) \Leftrightarrow \neg A \vee \neg B$
3. Use derivations to check each of the claims below; if a derivation indicates that a claim fails, present a counterexample that divides an open gap.
 - a. $A \vee B, A \Rightarrow \neg B$
 - b. $A \vee (B \wedge C) \Leftrightarrow (A \vee B) \wedge C$
 - c. $\neg (A \vee B) \Leftrightarrow \neg A \vee \neg B$