4.1.3. Disjunction in English

Once we set aside controversies about the meaning of *or*, there are few special problems that arise in analyzing sentences as disjunctions. Of course, we must continue to be careful that the components we identify are independent sentences and really may be combined by disjunction to capture the content of the original sentence. This can keep us from analyzing a sentence as a disjunction even though it contains the word *or*. For example, *Everyone stood at either the port or the starboard railing* may not be analyzed as *Everyone stood at the port railing* ∨ *everyone stood at the starboard railing*.

The word *or* may be used in English to join a series of items and our approach to such serial disjunctions will similar to that used for serial conjunctions. We need to use two disjunctions and impose some grouping, but it will not matter which disjunction we take to have the wider scope. The parentheses indicating the grouping we impose may be suppressed when an analysis is written—so *Al will visit England, France, or Germany* could be analyzed using a run-on disjunction as

Al will visit England ∨ *Al will visit France* ∨ *Al will visit Germany*

However, we must recognize the grouping again in order to apply laws of entailment stated for two-component disjunctions.

There are few stylistic variants of *or* in English, but there is one especially clear way of stating an inclusive disjunction that deserves some comment. We might avoid any suggestion that Al will not visit both France and Germany by restating our earlier example as follows.

Al will visit at least one of France and Germany.

That we can have any chance at all of avoiding the implicature requires some explanation because, even though conversational implicatures are not part of the content of what we say, they derive from it. So it is hard to avoid them (in a given conversational context) by saying the same thing in different words. Perhaps we succeed in the case at hand because the phrase *at least one* is slightly stilted and would be appropriate only if the simpler form *or* could not be used. The stilted language could provide a clue to the audience that the speaker wants to avoid the implicatures ordinarily carried by a disjunction, and the implicature that is carried by the content of the assertion would then end up being canceled by the way it was expressed.

The phrase *at least one* seems stilted in part because it presents a simple disjunction as if it was chosen from a whole family of similar claims, each saying something about how many alternatives from a list are true. For example, we might say that Al will not visit both countries by means of the following:

Al will visit at most one of France and Germany.

And we could state an exclusive disjunction as follows:

Al will visit exactly one of France and Germany.

Notice that this last sentence can be analyzed as the conjunction of the two preceding it.

With a list of more than two alternatives, there is a greater variety of claims of this sort; but, like the examples above, all of them can be expressed quite directly using conjunction, negation, and disjunction. For example, let us try to express the following sentence as a compound of the three abbreviated below it:

Exactly two of Dan, Ed, and Fred will make the finals

D: *Dan will make the finals*;
E: *Ed will make the finals*;
F: *Fred will make the finals*

As a first step in analyzing this sentence, we may note that it can be regarded as a conjunction of two claims, one saying that at least two of the three will make it and the other saying that at most two will. A claim that at most two will make it denies that all three will make it and can be expressed as $\neg$ (D $\wedge$ E $\wedge$ F). The claim that at least two will make it tells us that there is at least one true sentence of the form *a and b will make the finals* where *a* and *b* are different names chosen from among *Dan*, *Ed*, and *Fred*. Now there are three non-equivalent sentences of this form—namely, D $\wedge$ E, D $\wedge$ F, and E $\wedge$ F—so what we wish to say is that at least one of these three sentences is true. This can be expressed by the run-on disjunction (D $\wedge$ E) $\vee$ (D $\wedge$ F) $\vee$ (E $\wedge$ F). Putting the two analyses together, we get the sentence below as an analysis of the claim that exactly two will make it.

$$((D \wedge E) \vee (D \wedge F) \vee (E \wedge F)) \wedge \neg (D \wedge E \wedge F).$$

This analysis is admittedly complex, and no one would choose to carry it out for even a moderately long list of alternatives; but the fact that it would be theoretically possible to do so is interesting, for it shows that we can understand some implications that seem to depend on numerical reasoning—for example, the validity of

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solely in terms of the logical properties of *and*, *or*, and *not*. In 8.3.2, we will see that this idea can be carried further by using other logical constants. The possibility of understanding numerical reasoning as an aspect of purely logical reasoning was one of the key reasons for Frege's interest in logic and one of the chief motivations for its development at the end of the 19th and beginning of the 20th centuries.

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