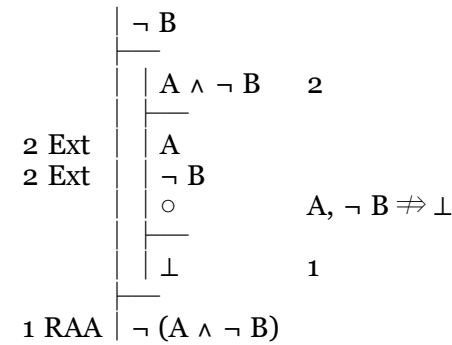


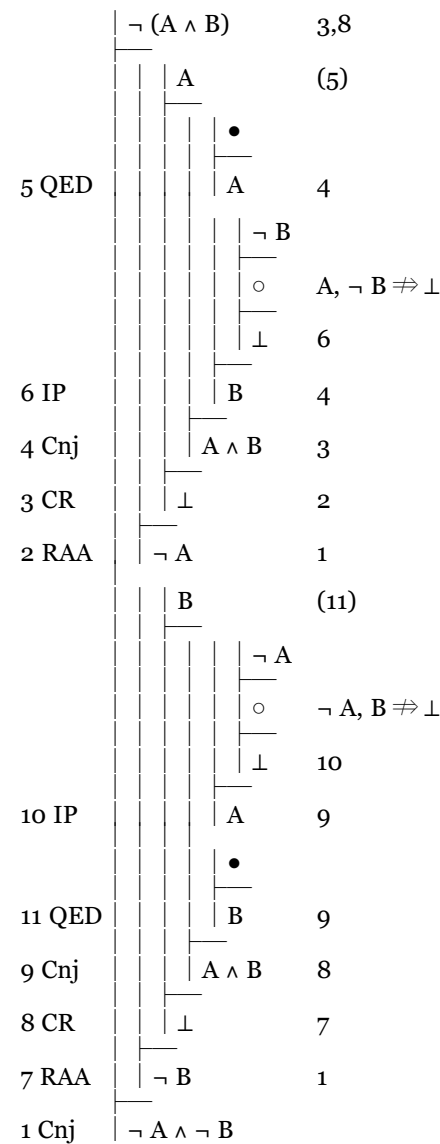
3.4.xa. Exercise answers

1. a.



A	B	$\neg B$	$\neg (A \wedge \neg B)$
T	F	Ⓣ	Ⓣ
T	F	Ⓣ	Ⓣ
F	T	Ⓣ	Ⓣ

b.



A	B	$\neg (A \wedge B)$	$\neg A \wedge \neg B$
T	F	Ⓣ	F
F	T	Ⓣ	F
F	F	Ⓣ	T
T	T	Ⓣ	F

divides the first dead-end gap
divides the second dead-end gap

c.

	$\neg(A \wedge B)$	3																																																			
	$\neg(B \wedge C)$	7																																																			
	$A \wedge C$	2																																																			
2 Ext	A																																																				
2 Ext	C	(10)																																																			
	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">•</td> <td></td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">A</td> <td style="padding-left: 10px;">4</td> </tr> </table>	•		A	4																																																
•																																																					
A	4																																																				
5 QED	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;"> <table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\neg B$</td> <td></td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 5px;"> <table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\neg B$</td> <td></td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">○</td> <td style="padding-left: 10px;">A, $\neg B, C \not\Rightarrow \perp$</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">\perp</td> <td style="padding-left: 10px;">9</td> </tr> </table> </td> <td style="padding-left: 10px;">8</td> </tr> </table> </td> <td></td> </tr> <tr> <td style="padding-right: 10px;">9 IP</td> <td style="border-left: 1px solid black; padding-left: 5px;">B</td> <td style="padding-left: 10px;">8</td> </tr> <tr> <td></td> <td style="border-left: 1px solid black; 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T F T	Ⓣ F Ⓣ F Ⓣ T																																																				

2. a.

	$\neg(A \wedge \neg B)$	2						
	$\neg B$	(5)						
	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\neg A$</td> <td></td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">○</td> <td style="padding-left: 10px;">$\neg A, \neg B \not\Rightarrow \perp$</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">\perp</td> <td style="padding-left: 10px;">4</td> </tr> </table>	$\neg A$		○	$\neg A, \neg B \not\Rightarrow \perp$	\perp	4	
$\neg A$								
○	$\neg A, \neg B \not\Rightarrow \perp$							
\perp	4							
4 IP	A	3						
	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">•</td> <td></td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">$\neg B$</td> <td style="padding-left: 10px;">3</td> </tr> </table>	•		$\neg B$	3			
•								
$\neg B$	3							
5 QED	$A \wedge \neg B$	2						
3 Cnj	\perp	1						
2 CR	B							
1 RAA	$\neg(A \wedge \neg B) / B$							
	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">A B</td> <td style="padding-left: 10px;">$\neg(A \wedge \neg B) / B$</td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">F F</td> <td style="padding-left: 10px;">Ⓣ F T Ⓣ</td> </tr> </table>	A B	$\neg(A \wedge \neg B) / B$	F F	Ⓣ F T Ⓣ			
A B	$\neg(A \wedge \neg B) / B$							
F F	Ⓣ F T Ⓣ							

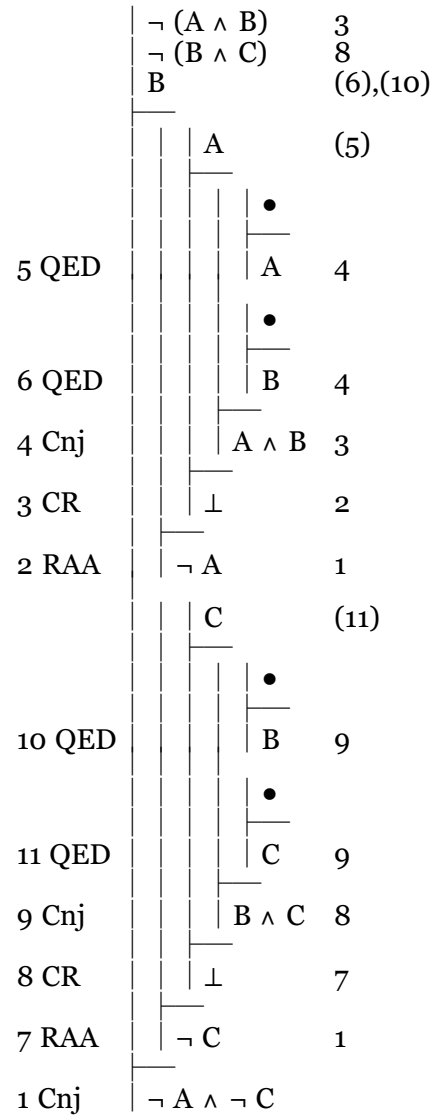
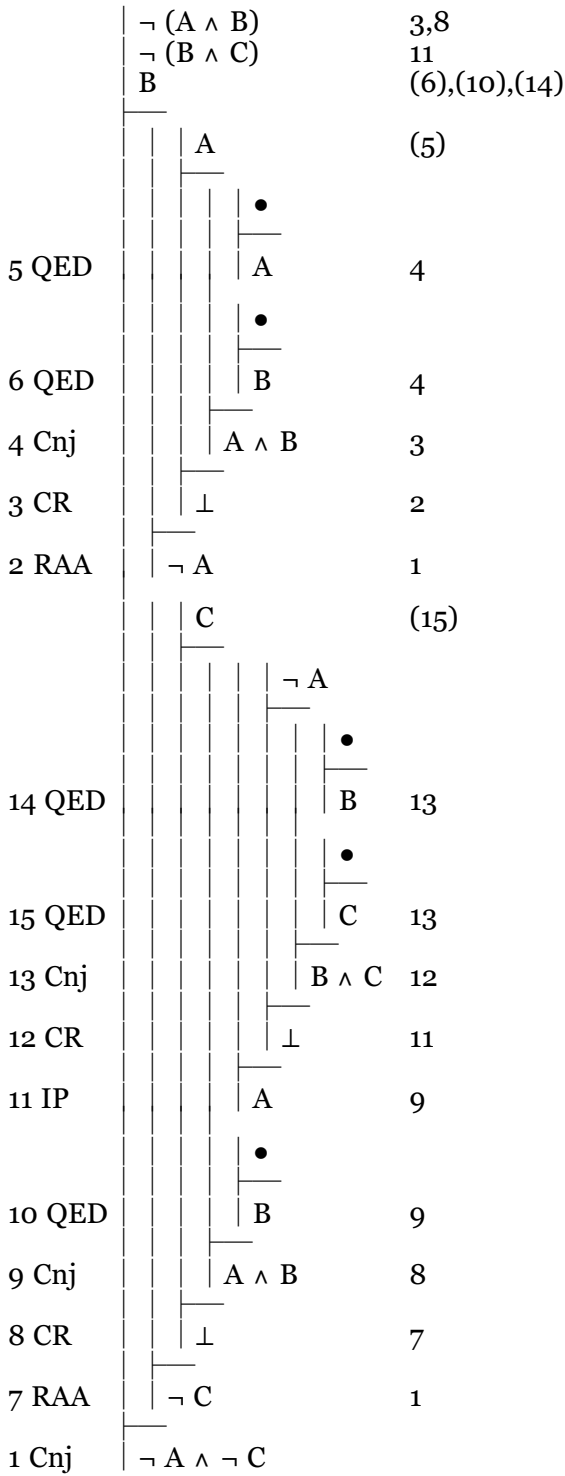
b.

	$\neg (A \wedge B)$	3										
	$B \wedge A$	2										
2 Ext	B	(6)										
2 Ext	A	(5)										
	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;"> <table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">•</td> <td style="padding-left: 5px;"></td> </tr> <tr> <td style="border-left: 1px solid black; border-bottom: 1px solid black; padding-left: 5px;">A</td> <td style="padding-left: 5px;">4</td> </tr> </table> </td> <td style="padding-left: 5px;"></td> </tr> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">•</td> <td style="padding-left: 5px;"></td> </tr> <tr> <td style="border-left: 1px solid black; border-bottom: 1px solid black; padding-left: 5px;">B</td> <td style="padding-left: 5px;">4</td> </tr> </table>	<table style="border-collapse: collapse; margin-left: 5px;"> <tr> <td style="border-left: 1px solid black; padding-left: 5px;">•</td> <td style="padding-left: 5px;"></td> </tr> <tr> <td style="border-left: 1px solid black; border-bottom: 1px solid black; padding-left: 5px;">A</td> <td style="padding-left: 5px;">4</td> </tr> </table>	•		A	4		•		B	4	
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•												
A	4											
•												
B	4											
5 QED	$A \wedge B$	3										
4 Cnj	\perp	1										
3 CR	$\neg (B \wedge A)$											
1 RAA												

c.

	$\neg (A \wedge \neg B)$	3																																								
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T T	Ⓣ																																									

d.



The derivation on the left exploits resources in their order of appearance; while the one above chooses, at stage 8, the resource that is most closely connected with other resources of the gap in which it is exploited. Notice that derivation on the left is eventually led to exploit the same resource to the same effect.

e.

	$\neg (A \wedge \neg (B \wedge \neg (C \wedge \neg D)))$	3	
	$A \wedge \neg (B \wedge D)$	2	
2 Ext	A	(5)	
2 Ext	$\neg (B \wedge D)$	8	
	•		
5 QED	A	4	
	$B \wedge \neg (C \wedge \neg D)$	7	
7 Ext	B	(10)	
7 Ext	$\neg (C \wedge \neg D)$	12	
	•		
10 QED	B	9	
	$\neg D$	(15)	
	$\neg C$		
	○		$A, B, \neg C, \neg D \not\Rightarrow \perp$
	⊥	14	
14 IP	C	13	
	•		
15 QED	$\neg D$	13	
13	$C \wedge \neg D$	12	
12 CR	⊥	11	
11 IP	D	9	
9 Cnj	$B \wedge D$	8	
8 CR	⊥	6	
6 RAA	$\neg (B \wedge \neg (C \wedge \neg D))$	4	
4 Cnj	$A \wedge \neg (B \wedge \neg (C \wedge \neg D))$	3	
3 CR	⊥	1	
1 RAA	$\neg (A \wedge \neg (B \wedge D))$		
	$\neg (A \wedge \neg (B \wedge \neg (C \wedge \neg D)))$		$\neg (A \wedge \neg (B \wedge D))$
	T T F F		Ⓓ F F T T F T Ⓔ T T F