

3.4.s. Summary

The adequacy of our current system is established by showing that it is sufficient, conservative, and decisive. The arguments for sufficiency and decisiveness take a slightly different form from those used in the last chapter. A gap that remains open at a dead end will now always have \perp as its goal and its resources are limited to \top , atomic sentences, and negated atomic sentences, with no resource being the negation of another. Any such gap can be divided by an interpretation that makes all its active resources true, so the rules are sufficient to close any gap that cannot be divided. Also, we can show that our new rules will not lead us on forever by showing that they are progressive by leading us always to replace goals or resources by others of a lower `grade` eventually leading us to goals and resources that are `minimal`, a class that includes \top , atomic sentences and negated atomic sentences in the case of resources and \perp alone in the case of goals.

Dead-end gaps will now have proximate arguments that are *reductios*, so the failure of a derivation will turn on the failure of a *reductio* and thus on the fact that the premises of the *reductio* form a consistent set. Thus any example of the failure of entailment will henceforth be due to the consistency of some set.