3.2.3. Some examples

Here is a derivation that uses the rules RAA and NC:

```
            │ A ∧ ¬ C              1
            ├───
   1 Ext    │ A
   1 Ext    │ ¬ C                 (5)
            │  │ B ∧ (C ∧ ¬ D)     3
            │  ├───
   3 Ext    │  │ B
   3 Ext    │  │ C ∧ ¬ D           4
   4 Ext    │  │ C                (5)
   4 Ext    │  │ ¬ D
            │  │ •
            │  ├───
   5 NC     │  │ ⊥                 2
            ├───
   2 RAA    │ ¬ (B ∧ (C ∧ ¬ D))
```

One feature of this derivation will now be typical: it is possible to have all uses of Ext at the beginning of the derivation since some of them are used to exploit the supposition B ∧ (C ∧ ¬ D)). Of course, we might have used Reductio Ad Absurdum and made this supposition at the first stage and then applied Ext to all the resources that had accumulated. But the following derivation shows that even this degree of grouping will not always be possible.

```
            │ A ∧ ¬ B             1
            ├───
   1 Ext    │ A                  (4)
   1 Ext    │ ¬ B                (6)
            │  │ •
            │  ├───
   4 QED    │  │ A                2
            │  │  │ B ∧ C          5
            │  │  ├───
   5 Ext    │  │  │ B             (6)
   5 Ext    │  │  │ C
            │  │  │ •
            │  │  ├───
   6 NC     │  │  │ ⊥              3
            │  │  ├───
   3 RAA    │  │ ¬ (B ∧ C)        2
            │  ├───
   2 Cnj    │ A ∧ ¬ (B ∧ C)
```

We might have waited until after the supposition B ∧ C was made before applying Ext but, by then, there would be two gaps and the first premise would have to be exploited in each in order for them to close. In general, it is wise (though not necessary) to apply Ext to a conjunction as soon as it appears as a resource, but conjunctions may continue to appear as resources from time to time as a derivation develops.

Now let's look at the sort of derivation we might give for the argument that began §3.2.2 . We can analyze the first premise of that argument as follows:

Ann and Bill were not both home without the car being in the driveway
¬ Ann and Bill were both home without the car being in the driveway
¬ (Ann and Bill were both home ∧ ¬ the car was in the driveway)
¬ ((Ann was home ∧ Bill was home) ∧ ¬ the car was in the driveway)

¬ ((A ∧ B) ∧ ¬ C)
not both both A and B and not C

[A; *Ann was home*; B: *Bill was home*; C: *the car was in the driveway*]

So the full argument takes the form:

$$\frac{\begin{array}{c}\neg((A \wedge B) \wedge \neg C) \\ A \wedge \neg C\end{array}}{\neg B}$$

The negative first premise is crucial for the argument, but we have no way of using it at the moment without having the compound it negates as a resource. To get that compound—i.e., (A ∧ B) ∧ ¬ C—as a resource, we need to use Adjunction to add its first conjunct and the full compound.

```
            │ ¬ ((A ∧ B) ∧ ¬ C)   (6)
            │ A ∧ ¬ C             2
            ├──
    2 Ext   │ A                   (4)
    2 Ext   │ ¬ C                 (5)

            │ │ B                 (4)
            │ ├──
    4 Adj   │ │ A ∧ B             X,(5)
    5 Adj   │ │ (A ∧ B) ∧ ¬ C     X,(6)
            │ │ •
            │ ├──
    6 NC    │ │ ⊥                 3
            ├──
    3 RAA   │ ¬ B
```

The need to use Adjunction in cases like this will end when we get the further rules of the next section, but it will sometimes still be a natural approach to establishing an entailment.

Now let's see what the derivation looks like if we replace the symbolic sentences by the actual English sentences they analyze:

```
            │ Ann and Bill were not both home without the car being in the driveway   (6)
            │ Ann was home but the car was not in the driveway                        2
            ├──
    2 Ext   │ Ann was home                                                            (4)
    2 Ext   │ the car was not in the driveway                                         (5)

            │ │ Bill was home                                                         (4)
            │ ├──
    4 Adj   │ │ Ann and Bill were both home                                           X,(5)
    5 Adj   │ │ Ann and Bill were both home without the car being in the driveway     X,(6)
            │ │ •
            │ ├──
    6 NC    │ │ ⊥                                                                      3
            ├──
    3 RAA   │ Bill was not home
```

In a stretch of explicit deductive argumentation in English, various sorts of connecting

language would be used to get the effect of the lines and annotations that structure the derivation. This is not the sort of entailment where such an explicit argument would ordinarily be given, but if one were offered, it might run something like this:

> We assume that **Ann and Bill were not both home without the car being in the driveway** and also that **Ann was home but the car was not in the driveway**. So we know that **Ann was home**. And we also know that **the car was not in the driveway**.
>
> Now suppose (for the sake of reductio) that **Bill was home**. It would follow that **Ann and Bill were both home**. And then we would know that **Ann and Bill were both home without the car being in the driveway**. But that contradicts one of our initial assumptions.
>
> So we can conclude that **Bill was not home**.

The modal verb *would* has been used here in the *reductio* argument of the second paragraph to emphasize that the situation being described need not be a real one. It is possible to go further in that direction by phrasing the supposition itself as *Suppose that Bill were home*; but it is also possible to let the verb *suppose* suffice to show that what follows is not a consequence of the initial premises.

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