

### 3.2.1. The duality of premises and alternatives

The [basic law for relative exhaustiveness](#) tells us that, when sentences  $\phi$  and  $\phi'$  are contradictory, having one as a premise comes to the same thing as having the other as a conclusion—that is,

$$\Gamma \Rightarrow \phi, \Delta \text{ if and only if } \Gamma, \phi' \Rightarrow \Delta$$

If we apply this to the contradictories  $\phi$  and  $\neg \phi$ , we get a pair of principles

$$\begin{aligned} \Gamma \Rightarrow \neg \phi, \Delta \text{ if and only if } \Gamma, \phi \Rightarrow \Delta \\ \Gamma, \neg \phi \Rightarrow \Delta \text{ if and only if } \Gamma \Rightarrow \phi, \Delta \end{aligned}$$

where we get the second by turning around both the pair of contradictories and the *if-and-only-if* claim. That is, having a negation as either a premise or alternative comes to the same thing as having the unnegated sentence in the opposite role. The kind of opposition in question here is the sort of duality mentioned in [1.4.6](#).

We do not study relative exhaustiveness directly, and one use of the basic law for relative exhaustiveness is to exchange alternatives for premises so that a claim of relative exhaustiveness may be converted into a claim of entailment. But suppose we begin with only a single alternative; that is, suppose  $\Delta$  is empty. In this case, when  $\phi$  and  $\phi'$  are contradictory, we can say that

$$\Gamma \Rightarrow \phi \text{ if and only if } \Gamma, \phi' \Rightarrow$$

where the right-hand side says that  $\phi'$  is inconsistent with (or is excluded by)  $\Gamma$ . When we express inconsistency by way of the validity of a *reductio* argument, we get the following **basic law for contradictories**:

$$\text{if } \phi \text{ and } \phi' \text{ are contradictory, then } \Gamma \Rightarrow \phi \text{ if and only if } \Gamma, \phi' \Rightarrow \perp$$

Indeed, if the right-hand side of this holds for every choice of the set  $\Gamma$ , then  $\phi$  and  $\phi'$  must be contradictory. (To see why, think what follows if  $\Gamma$  is chosen first as the single sentence  $\phi$  and then as the sentence  $\neg \phi'$ .)

Now we can get some basic principles for negation with regard to entailment by applying the basic law for contradictories to the case of negation in the way we applied the basic law for relative exhaustiveness above. That is, if we take  $\phi$  and  $\neg \phi$  as our contradictory sentences, we get:

**Law for negation as a conclusion.**  $\Gamma \Rightarrow \neg \phi$  if and only if  $\Gamma, \phi \Rightarrow \perp$

**Law for negation as a premise.**  $\Gamma, \neg \phi \Rightarrow \perp$  if and only if  $\Gamma \Rightarrow \phi$

again reversing the pair of contradictories and the *if-and-only-if* claim to get the second.

Although these principles are dual in something like the way that the earlier pair for relative exhaustiveness were, each has a rather different significance. The first captures the core properties of negation while the second is closely tied to the equivalence of  $\neg \neg \phi$  with  $\phi$  (which, as was noted in §3.1.3, is about as controversial as anything gets in logic). Also, while the first will provide us with straightforward ways of planning for negative goals and carrying out these plans, the second gives an account of the role of negative premises only in the context of *reductio* arguments and, for this reason, has a less straightforward implementation as a derivation rule that we will postpone until §3.3.