

3.1.s. Summary

Negation is an operation associated with the English word *not*. It generates a compound sentence from a single component, so it is a connective that serves to modify a sentence rather than to combine sentences. The **not symbol** \neg is our notation for negation. As English notation for $\neg \phi$, we use **not ϕ** .

A sentence and its negation cannot be both true (they are **mutually exclusive**) and cannot be both false (they are **jointly exhaustive**); in short, they must have different truth values (they are **contradictory**). Each leaves open the possibilities the other rules out and rules out the possibilities the other leaves open. This means that negation, like **conjunction**, has a truth table; in other words it is a **truth-functional connective**. Not all connectives are truth-functional. **Truth-functional logic** is the branch of logic which studies those that are, but there are branches of logic—such as **tense logic** and **modal logic**—in which non-truth-functional connectives are studied.

Negation appears in English not only in connection with the word *not* but also with negative prefixes (though such a prefix does not always mark negation because it does not always produce a sentence that is contradictory to the original). Negation also appears with uses of *no* in phrases of the form *no X*, uses that can often be treated as the negation of *at least one* or *some*. The same sort of treatment is usually what is required when *not* appears along with the word *any* (which usually must be rephrased when *not* is removed). By negating a negation, we can produce a **double negation**, but this undoes the negation rather than generating a logical form with new properties.

The really new ideas come with the negation of conjunctions, but conjunctions whose components may involve negation also provide important forms of expression. A number of forms are shown below, with labels that suggest the sort of English sentences they serve to analyze:

not-both form	$\neg (\phi \wedge \psi)$	not both ϕ and ψ
not-but form	$\neg \phi \wedge \psi$	both not ϕ and ψ
but-not form	$\phi \wedge \neg \psi$	both ϕ and not ψ
not-and-not form	$\neg \phi \wedge \neg \psi$	both not ϕ and not ψ
not-without form	$\neg (\phi \wedge \neg \psi)$	not both ϕ and not ψ

That the last is the denial of the third reflects the fact that *without* can

be used to express a *but-not* form. Also *neither-nor* can be used to express a *not-and-not* form. More generally, negation and conjunction form a truth-functionally complete set of connectives in the sense that any truth-functional compound can be expressed using them alone.

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