3.1.2. Contradictory propositions

We could base the truth conditions of negation directly on the observation that the word *false* means '*not* true' and the word *true* means '*not* false.' But it will be more enlightening to base it instead on some understanding of the logical relations between a negation $\neg \phi$ (or `not` ϕ) and its component ϕ.

One obvious generalization about negation is that a negative sentence is incompatible with the component that is negated. For example, in the traditional children's story, even before sitting down to her taste test, Goldilocks knew that *The porridge is too hot* and *The porridge is not too hot* could not both describe the same bowl. Each excludes the other; they are mutually exclusive (in the sense defined in 1.4.4). We can explain this fact about negation if we assume that the negation $\neg \phi$ of a sentence ϕ is false whenever the sentence ϕ is true. And that settles the part of the truth table for negation shown below.

ϕ	$\neg \phi$
T	F

But it does not settle the rest. The sentences *The porridge is too hot* and *The porridge is too cold* are also mutually exclusive, but Goldilocks found two cases in which *The porridge is too hot* was false, one in which *The porridge is too cold* is true and another in which it was not. So the mutual exclusiveness of ϕ and $\neg \phi$ is not enough to settle the truth value of $\neg \phi$ when ϕ is false.

There is a second relation between a sentence and its negation that does settle this value. While the falsity of both *The porridge is too hot* and *The porridge is too cold* would leave open the possibility that the porridge is just right, *The porridge is too hot* and *The porridge is not too hot* allow no third case. That means the two sentences are jointly exhaustive of all possibilities (see 1.4.5 for this idea). This relation serves to settle the second row of the truth table for negation; if ϕ is false then $\neg \phi$ must be true.

ϕ	$\neg \phi$
T	F
F	T

A negation $\neg \phi$ thus has a truth value that is always the opposite of the truth value of its component ϕ. In 1.4.5, we spoke of such sentences (that is, sentences that are both mutually exclusive and jointly exhaustive) as "contradictory." So a sentence and its negation are contradictory sentences; each contradicts the other. The negation of a sentence ϕ need not be the only sentence that contradicts ϕ, but any sentence that stands in this relation to ϕ will be logically equivalent to $\neg \phi$.

Figure 3.1.2-1 shows the effect of negation on the proposition expressed; the possibilities ruled out by the sentence (A) and its negation (B) are hatched with diagonal lines. The images of dice recall the example of Figure 2.1.1-1 ; if they are taken to indicate regions consisting of the possible worlds in which a certain die shows one or another number, the proposition shown in 3.1.2-1A is *The number shown by the die is less than 4* and 3.1.2-1B illustrates its negation.

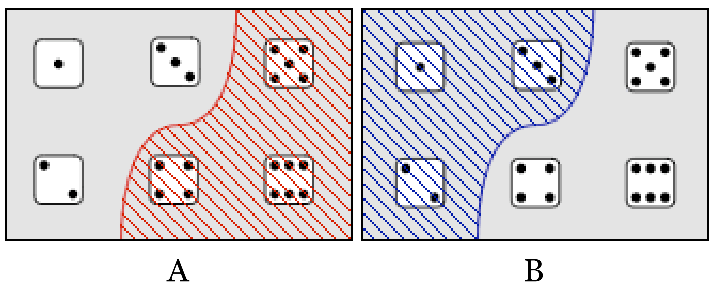

Fig. 3.1.2-1. Propositions expressed by a sentence (A) and its negation (B).

The possibilities left open by a sentence are ruled out by its negation—no possibilities are left open by both—because the two are mutually exclusive. And the possibilities ruled out by a sentence are left open by a sentence—none are ruled out by both—because the two are jointly exhaustive.

Connectives that have truth tables express truth functions and are therefore said to be ***truth functional***. We have seen that conjunction and negation are truth functional, but not all connectives have this property. The following simple example of a non-truth-functional connective that should suggest a whole range of further examples. Compare these two sentences:

The bridge is not finished
The bridge will never be finished.

The truth value of the first is determined once we know the truth value of *The bridge is finished*, but this is not always enough information in the case of the second. When *The bridge is finished* is true, we know that *The bridge will never be finished* is false; but, when *The bridge is finished* is false, we need more information to determine the truth value of *The bridge will never be finished*. In particular, we need at least some information about the truth value of *The bridge is finished* at times in the future; and before we can know that *The bridge will never be* *finished* is true, we need to know the truth value of *The bridge is finished* at *all* times in the future. And this means that the connective marked by the English form *It will never be the case that* ϕ is not truth functional.

We will limit our study of connectives to those that are truth-functional. The study of such connectives is ***truth-functional logic*** (a phrase that was mentioned in 1.1.5). The connective expressed by *It will never be the case that* ϕ would be studied by ***tense logic***, the logic of tenses and other temporal modifiers. This is one part of the logic of connectives that lies beyond truth-functional logic. Another part is the logic of modal auxiliaries like *must* and *can*. These, too, are associated with non-truth-functional connectives, and the study of the logical properties of these connectives is referred to as ***modal logic***, an ancient branch of logic that became an active area of research again in the 20^{th} century.