

2.4.s. Summary

The introduction of a lemma is one way of dividing up the work of a proof. We can implement this idea in derivations by dividing a gap into two, with one child having the lemma as a goal and the other having it as a further assumption to use in reaching the goal of the parent gap. The rule [Lemma \(Lem\)](#) that does this is not safe in general nor is it always progressive, and we will use only special instances of it.

A lemma is always safe when it is entailed by the goal it is designed to help us reach. The principal use of this idea will come in arguments whose goal is \perp —that is, in [reductio arguments](#). Since \perp entails any sentence a rule [Lemma for Reductio \(LFR\)](#), which allows free use of lemmas in *reductio* arguments will be safe (though some restriction on its use is needed to insure it is progressive).

A lemma is also safe if it is entailed by things we already know. Rules applying this idea will be designed for particular sorts of entailment and, since such a lemma is known to follow from our resources, there is no need to divide the gap or even introduce a new scope line. Indeed, we will use this sort of lemma only in [attachment rules](#) that add the lemma as an available but inactive resource. The first example of this sort of rule is [Adjunction \(Adj\)](#) which adds a conjunction when both conjuncts are already available. Although attachment rules can help us to close gaps sooner, the rules themselves are not direct, and some care is needed in their use if they are to be progressive.

The derivation rules we have so far are summarized in the table below. The names of the rules are links to the point in the text where they were initially described; look there to see the actual form taken by the rule.

<i>Rules for developing gaps</i>		
	<i>for resources</i>	<i>for goals</i>

conjunction $\phi \wedge \psi$	Ext	Cnj
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<i>Rules for closing gaps</i>	
<i>when to close</i>	<i>rule</i>

the goal is also a resource	QED
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\top is the goal	ENV
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\perp is a resource	EFQ
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Basic system

<i>Attachment rule</i>	
<i>added resource</i>	<i>rule</i>

$\phi \wedge \psi$	Adj
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Added rules (optional)