

2.4.3. Attachment rules

When discussing the minimal soundness of QED in [2.3.2](#) we saw that it would be legitimate for a rule to close a gap whenever its goal is entailed by active resources. (If that legitimacy seems obvious, remember that such a rule is only *minimally* sound because it can enable us to sidestep the consequences of unsafe rules used earlier in the derivation.) We will not employ such a sweeping rule but we will extend the use of QED (and later rules which use inactive resources) by rules which add new resources to those available in a gap without changing either exploiting active resources or analyzing the goal. The first example of such a rule is the following way of developing a gap, which we will call **Adjunction**:

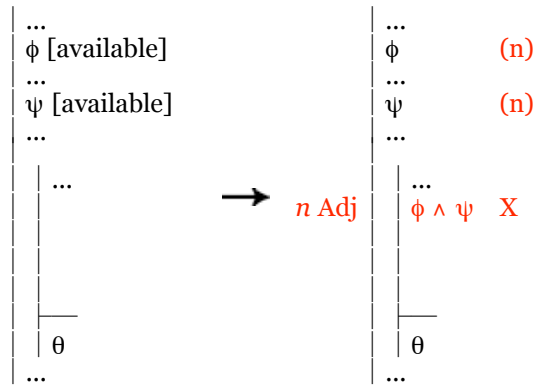


Fig. 2.4.3-1. Developing a derivation by applying Adj at stage n .

The added conjunction functions as a lemma, but this rule represents a way of using lemmas that has a number of special features. Notice that the lemma $\phi \wedge \psi$ does not lie to the right of a new scope line, as it does in the second gap introduced by LFR. There are two reasons for this. First, we have not branched the gap so the added resource, so there is no second gap to set outside the scope of the new resource. More importantly, we do not need to mark this new resource off as an added assumption made for a part of the derivation because it is entailed by resources already available throughout the gap we are developing.

Notice also that we treat this rule not as a way to plan for our goal but simply as a way to add resources. It does not exploit resources in order to add others, however, and the stage numbers to the right of the premises are parenthesized to indicate that they have not been exploited.

The X to the right of $\phi \wedge \psi$ is intended to indicate that this resource need not be exploited further. One way to think about this is to suppose that $\phi \wedge \psi$ has been introduced as something already exploited. That is, although it need not have been a once active but exploited resource (and there would be no point in adding it if it was), it has a status similar to such resources. It is marked at its right as exploited resources are, but it is not marked by a stage number because it was not exploited at some previous stage.

Adjunction is one example of a group of rules we will refer to as **attachment rules**. Any such rule R will exhibit the following general pattern.

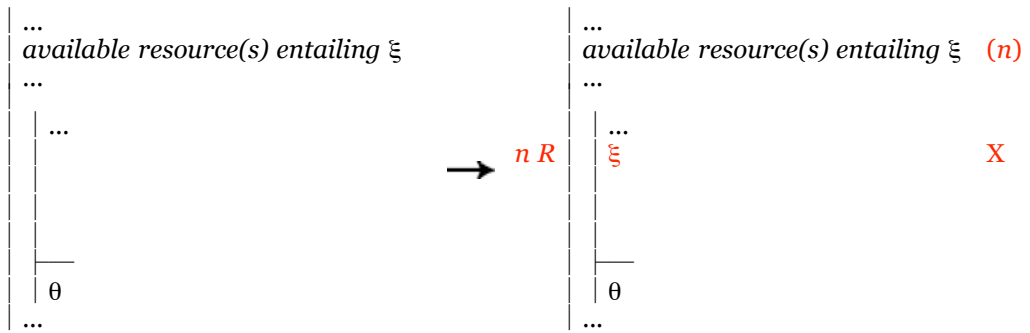


Fig. 2.4.3-2. Developing a derivation by applying an attachment rule R at stage n .

Since the added lemma does not become an active resource, the proximate argument of the child gap that is produced by an attachment rule is the same as the parent's proximate argument. As a result, safety and (utter) soundness hold just as they would for a gap that was completely unchanged. The only question about such a rule concerns its impact on the (minimal) soundness of rules like QED that use merely available resources. But any resource made available by attachment rule will be entailed by the active resources of the gap together with the active resources of its ancestors. Any interpretation that divides a gap and all its ancestors will then make such an resource true; and this will be enough for us to insure the soundness of rules using available resources. (To rehearse the argument for the case of QED again: if the goal of a gap is among its available resources, no interpretation can divide both it and all its ancestors because, to do this, an interpretation would need to make the goal false while making not only the active resources but also all other available resources true, and this is impossible when the goal is among the available resources. The key to this argument is being sure that all available resources are true whenever the active resources of a gap and all its ancestors are true, and attachment rules do not undermine our confidence in this.)

A rule like Adj clearly raises questions about decisiveness since the lemma it introduces is more complex than the premises it is based on. This increased complexity will be typical of attachment rules and is the reason for their name. A requirement that the lemma be a component of a goal or active resource would be a natural way of insuring decisiveness since such cases will represent the most valuable uses of attachment rules. Here, though, we need to remember that a sentence is a component of itself. Indeed, one common use of attachment rules will be to introduce the goal itself as an available resource in order to apply QED and close the gap. The following derivation is a simple example of this in the case of Adj.

	A \wedge B	1
	C	(4)
1 Ext	A	(3)
1 Ext	B	(4)
	•	
	A	2
3 QED	B \wedge C	X,(5)
	•	
	B \wedge C	2
5 QED	A \wedge (B \wedge C)	
2 Cnj		

With two uses of Cnj, we would not have needed Adj and, with two uses of Adj, we would not have needed Cnj; but it is the sort of mixed use of the two illustrated here that brings

us closest to typical patterns of explicit deductive argument.

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