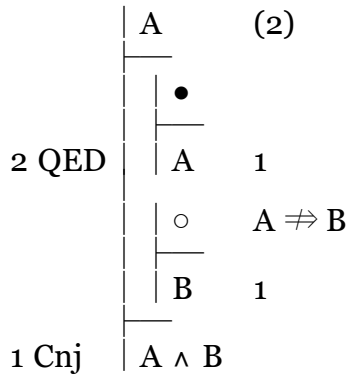


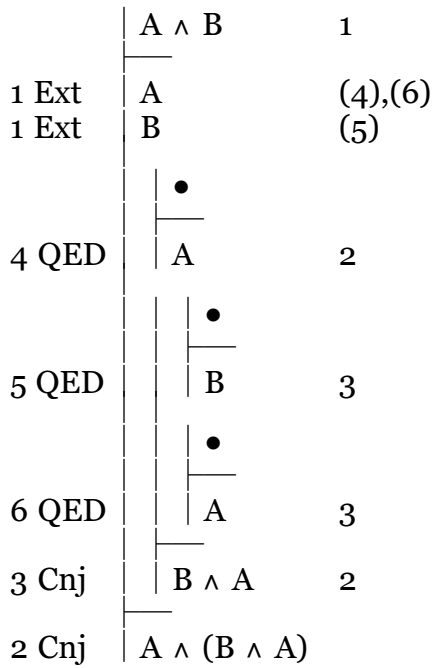
2.3.xa. Exercise answers

1.



A	B	$A / A \wedge B$	
T	F	Ⓓ	Ⓕ

2.



3.

	$B \wedge E$	1									
	$C \wedge \top$	2									
1 Ext	B	(5)									
1 Ext	E										
2 Ext	C	(7)									
2 Ext	\top										
	<table style="margin: 0 auto; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-right: 5px;">○</td> <td style="padding-left: 10px;">$B, C, E, \top \not\Rightarrow A$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="border-top: 1px solid black; padding-right: 5px;"></td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-right: 5px;">A</td> <td style="padding-left: 10px;">4</td> </tr> </table>		○	$B, C, E, \top \not\Rightarrow A$					A	4	
	○	$B, C, E, \top \not\Rightarrow A$									
	A	4									
	<table style="margin: 0 auto; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-right: 5px;">●</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="border-top: 1px solid black; padding-right: 5px;"></td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">5 QED</td> <td style="padding-right: 5px;">B</td> <td style="padding-left: 10px;">4</td> </tr> </table>		●					5 QED	B	4	
	●										
5 QED	B	4									
4 Cnj	$A \wedge B$	3									
	<table style="margin: 0 auto; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-right: 5px;">●</td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="border-top: 1px solid black; padding-right: 5px;"></td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;">7 QED</td> <td style="padding-right: 5px;">C</td> <td style="padding-left: 10px;">6</td> </tr> </table>		●					7 QED	C	6	
	●										
7 QED	C	6									
	<table style="margin: 0 auto; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-right: 5px;">○</td> <td style="padding-left: 10px;">$B, C, E, \top \not\Rightarrow D$</td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="border-top: 1px solid black; padding-right: 5px;"></td> <td></td> </tr> <tr> <td style="border-right: 1px solid black; padding-right: 5px;"></td> <td style="padding-right: 5px;">D</td> <td style="padding-left: 10px;">6</td> </tr> </table>		○	$B, C, E, \top \not\Rightarrow D$					D	6	
	○	$B, C, E, \top \not\Rightarrow D$									
	D	6									
6 Cnj	$C \wedge D$	3									
3 Cnj	$(A \wedge B) \wedge (C \wedge D)$										

A	B	C	D	E	$B \wedge E, C \wedge \top$	$(A \wedge B) \wedge (C \wedge D)$
F	T	T	F	T	Ⓣ	Ⓣ
					Ⓣ	Ⓣ
					F	Ⓣ
					F	F

The derivation could have been ended after stage 4 when the first open gap has reached a dead end. Often answers will show a derivation continued further than necessary in order to show how the further steps would have worked out. The counterexample presented here divides both dead-end gaps; there are others that divide one of the two. Notice that \top is not assigned a value at the left of the table. Since its value is fixed by the stipulation that it is a tautology, a value need not and cannot be assigned to it as part of an extensional interpretation.

4.

	$A \wedge B$	1
	$B \wedge C$	2
	$B \wedge D$	3
1 Ext	A	(5)
1 Ext	B	
2 Ext	B	
2 Ext	C	
3 Ext	B	
3 Ext	D	(6)
•		
5 QED	A	4
•		
6 QED	D	4
4 Cnj	$A \wedge D$	

Clearly, there is redundancy in the active resources of the gaps after stage 3. Since both gaps close, the exploitation of the second premise at stage 2 is not necessary (though it would be necessary before any gap could reach a dead end). It would be possible to state rules so that the resource B was not repeated at stages 2 and 3, but such repetition does not ordinarily enlarge derivations significantly and makes it easier to check whether rules have been applied fully and correctly.

5.

	A	(6)
	B \wedge A	1
	D	(7)
1 Ext	B	(5)
1 Ext	A	
	•	
5 QED	B	2
	○	A,B,D \Rightarrow C
6 QED	•	4
4 Cnj	C \wedge A	3
	•	
7 QED	D	3
3 Cnj	(C \wedge A) \wedge D	2
2 Cnj	B \wedge ((C \wedge A) \wedge D)	
	<u>A B C D</u> <u>A, B \wedge A, D / B \wedge ((C \wedge A) \wedge D)</u>	
	T T F T (T) (T) (T) (F) F F	