

1.4.x. Exercise questions

1. Restate each of the following claims about logical properties and relations, putting into symbolic notation those stated in English and into English those stated in symbolic notation:
 - a. $\phi, \psi \Rightarrow \chi$
 - b. ϕ is entailed by ψ
 - c. $\phi \Leftrightarrow \phi$
 - d. $\psi \Rightarrow$
 - e. ϕ is inconsistent with Γ
 - f. ϕ is entailed by the members of Γ together with ψ

2. The following steps lead you to construct a proof of the law for lemmas

if $\Gamma, \phi \Rightarrow \psi$ and $\Gamma \Rightarrow \phi$, then $\Gamma \Rightarrow \psi$

Begin by supposing that $\Gamma, \phi \Rightarrow \psi$ and $\Gamma \Rightarrow \phi$ are both true. We want to show that, under this supposition, $\Gamma \Rightarrow \psi$ is also true. To do that, we consider any possible world w in which all members of Γ are true and try to show that ψ is true in w .

- a. Our supposition that $\Gamma, \phi \Rightarrow \psi$ and $\Gamma \Rightarrow \phi$ are both true combined with what we know about w enables us to conclude that ϕ is true. Why?
- b. Adding the information that ϕ is true in Γ to what we already knew, we can conclude that ψ is true. Why?

So, knowing about w only that all members of Γ were true, we are able to conclude that ψ is true. And that shows us that ψ is true in every world in which all members of Γ are true, which means that $\Gamma \Rightarrow \psi$.

Another approach to proving the law is to show that $\Gamma \Rightarrow \psi$ fails only if at least one of $\Gamma, \phi \Rightarrow \psi$ and $\Gamma \Rightarrow \phi$ fails. The following three steps show this:

- c. Suppose that w is a counterexample to $\Gamma \Rightarrow \psi$. What truth values do ψ and the members of Γ have in w ?
- d. What truth values are needed to have a counterexample to $\Gamma \Rightarrow \psi$? To have a counterexample to $\Gamma, \phi \Rightarrow \psi$?
- e. The world w from **c** will be a counterexample to either $\Gamma, \phi \Rightarrow \psi$ or $\Gamma \Rightarrow \phi$. Why?