

1.2.xa. Exercise answers

1.
 - a. The conclusion must be true; an argument with true premises and a false conclusion would not be valid.
 - b. Nothing can be said; some valid arguments whose premises are all false have true conclusions and others have false conclusions. (Here is an example of a valid argument with false premises and a true conclusion: *Indianapolis is the capital of Illinois, the capital of Illinois is east of the Wabash / Indianapolis is east of the Wabash.*)
 - c. Nothing can be said; some valid arguments with both true and false sentences among their premises have true conclusions and others have false conclusions.
2. You can say that the premises are not both true, that at least one of the two is false.
3.
 - a. Nothing; a pair of sentences with the same truth value may be equivalent but only if each has the same value as the other not only in the actual world but in all other possible worlds as well.
 - b. Nothing; a pair of sentences with the same truth value may be equivalent but only if each has the same value as the other in **all** possible worlds.
 - c. The sentences are not equivalent; equivalent sentences can never have different truth values.
 - d. Nothing; such sentences need not even have the same truth value. In any possible world in which χ is true, we know that ψ is true if ϕ is and that ϕ is true if ψ is. But ϕ and ψ might have different truth values in worlds where χ is false.
4.
 - a. ϕ expresses the same proposition as ψ so, like ψ , it leaves open all possible worlds and rules out none.
 - b. Nothing can be concluded about ϕ ; because \top cannot be false, it has no content and every sentence entails it.
 - c. Since ϕ is entailed by ψ , it must be true in every possible world in which ψ is true; therefore, ϕ must, like ψ , leave open all possibilities (that is, it is also a tautology)
 - d. ϕ expresses the same proposition as \perp so, like \perp , it rules out all possible worlds and leaves open none.
 - e. Since ϕ entails ψ , it must rule out every possibility that ψ rules out; but ψ rules out all possibilities so ϕ must as well (that is, it is also absurd).
 - f. Nothing can be concluded about ϕ ; because \perp rules out all possibilities, no sentence can have any further content and thus no sentence fails to be entailed by \perp .