1.1.x. Exercise questions

- 1. Assume that a statement of entailment $\Gamma\Rightarrow \phi$ holds when the premises Γ listed to the left of the arrow, taken together, contain all the information found in the conclusion ϕ displayed to its right. Using this understanding of entailment, decide for each of the following whether you can be sure that the statement is true (no matter what sentences are put in place of the Greek letters) and briefly explain your reasons. [In some cases a lower case Greek letter (our notation for a single sentence rather than a set) is used on the left of the sign \Rightarrow as shorthand for a set of premises with only a single member.]
 - **a.** $\phi \Rightarrow \phi$
 - **b.** if $\phi \Rightarrow \psi$ and $\psi \Rightarrow \chi$, then $\phi \Rightarrow \chi$
 - **c.** if $\phi \Rightarrow \psi$, then $\psi \Rightarrow \phi$
 - **d.** if (i) Γ , $\phi \Rightarrow \psi$ and (ii) $\Gamma \Rightarrow \phi$, then (iii) $\Gamma \Rightarrow \psi$ [Notice that this says that a premise ϕ of a valid argument Γ , ϕ / ψ may be dropped without destroying validity provided it is entailed by the remaining premises Γ .]
 - **e.** if χ , $\phi \Rightarrow \psi$ and χ , $\psi \Rightarrow \phi$, then ϕ , $\psi \Rightarrow \chi$
- 2. The basis for testing a scientific hypothesis can often be presented as an argument whose conclusion is a prediction about the result of the test and whose premises consist of the hypothesis being tested together with certain assumptions about the test (e.g., about the operation of any apparatus being used to perform the test).

$$\begin{array}{c} \text{hypothesis to be tested: } \textit{hypothesis} \\ \text{assumption about the test:} \left\{ \begin{matrix} assumption \\ ... \\ assumption \end{matrix} \right\} \text{premises} \end{array}$$

prediction of the test result: prediction conclusion

Suppose that the prediction is entailed by the hypothesis together with the assumptions about the test (i.e., suppose that the argument shown above is valid) and answer the following questions:

- **a.** Can you conclude that the hypothesis is true on the basis of a successful test (i.e., one whose result is as predicted)? Why or why not?
- **b.** Can you conclude that the hypothesis is false on the basis of an unsuccessful test (i.e., one whose result is not the one predicted)? Why or why not?

Glen Helman 01 Aug 2004