1.1.5. Formal logic

Another traditional label for our study is ***formal logic***. This term reflects another aspect of our study of reasoning. Even among the inferences that are deductive, we will consider only ones that do not depend on the *subject matter* of the data. This means that they will not depend on the concepts employed to describe particular subjects, but it also means that they will not depend the mathematical structures (numbers, shapes, etc.) that might be employed in such descriptions. We will limit ourselves to inferences that depend only on the *form* of the claims made in stating the data.

The distinction between form and content is a relative one and, while the use of mathematical methods to extract information will count as a concern with content when it is compared with the sort of inferences we will study, it can count as formal relative to other ways of extracting information. What matters for much of the numerical analysis of data is the numbers that appear in a body of measurements, not the nature of the quantities measured.

In a similar way, what matters for formal logic is the appearance of certain words or grammatical constructions that indicate the kind of claim that is being made by statements expressing the data. Examples of such logical words are *and*, *not*, *or*, *if*, *is* (in the sense of *is identical to*), *every*, and *some*. While this list is by no means exhaustive, it does provide a fair indication of the forms we will study. Indeed, these seven words could serve as title for the seven chapters that will follow this one. The way in which a statement is put together using such words (and using logically significant grammatical constructions not directly marked by words) is its ***logical form***, and formal logic is the study of reasoning whose quality rests on the logical forms of statements.

The norms of deductive reasoning that we will study will be general rules applying to all statements with certain logical forms. It happens that we can give an exhaustive account of such rules in the case of the logical forms that we will consider, so the content of the course can be defined by these forms. ***Truth-functional logic***, which will occupy us through chapter 5, is concerned with logical forms that can be expressed using the words *and*, *not*, *or*, and *if* while ***first-order logic (with identity)*** is concerned with the full list above, adding forms that can be expressed by the words *is*, *every*, and *some*.

Finally, we will take a quick look at the reason for the term ***symbolic logic*** that appears in the course title. Most of what this term indicates about the content of our study is captured already by the term ***formal***

logic because most of the symbols we use will serve to represent logical forms. Certain of the logical forms that appear in the study of truth-functional logic are analogous to patterns appearing in the symbolic statements of algebraic laws. Analogies of this sort were recognized by G. W. Leibniz (1646-1716) and by others after him, but they were first pursued extensively by George Boole (1815-1864), who adopted a notation for logic that was modeled after algebraic notation. The style of notation that is now standard among logicians owes something to Boole (though the individual symbols are different) and something also to the notation used by Gottlob Frege (1848-1925), who noted analogies between first-order logic and the mathematical theory of functions. This interest in analogies with mathematical theories distinguished logic as studied by Boole and Frege from its more traditional study, and the term *symbolic* has often been used to capture this distinction. The phrase ***mathematical logic*** would be equally appropriate and it has often been used in this way; but this label is also used more specifically for an application of logic to mathematics that takes the theories of mathematics as objects of mathematical study in their own right, a kind of research that is also known as ***metamathematics*** (which means, roughly, 'the mathematics of mathematics').

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