

Problem

(This problem is adapted from one in *Mathematics and Informatics Quarterly* 9:4 Dec 1999 p. 162.)

Some students are honest, others not. Also, some are sane while others are insane. Sane students' beliefs are true, while insane students' beliefs are false; honest students say things they *believe* are true, while dishonest students say things they believe are false. (For example, a dishonest insane student will believe that Wabash is a women's college and say that it is not.) Overheard at my office hours:

Will: "I am insane."

Xerxes: "I am honest."

Yakov: "I am dishonest."

Zhi: "I am sane."

Will: "Yakov is honest."

Xerxes: "Zhi is insane."

Yakov: "Xerxes is dishonest."

Zhi: "Yakov is sane."

Who is sane and insane, and who is honest and dishonest?

Solution

Let us first look at the implications of the first four statements. No student believes he is insane (because insane students will believe, insantly, that they are sane), so Will must be dishonest. If Xerxes is honest, then he must believe he is honest, in which case he is also sane; if Xerxes is dishonest, then he must believe he is dishonest, in which case he again is sane; so Xerxes is sane. Any sane student will say that he is honest if he is honest, and say that he is honest if he is dishonest; so Yakov must be insane. A dishonest sane student will say he is insane, and a dishonest insane student, believing himself to be sane, will say he is insane. So Zhi must be honest.

We now examine the last four statements. We know Zhi is honest and that Yakov is insane, so from Zhi's statement we can conclude that Zhi is sane. This makes Xerxes' statement true; already knowing that he is sane, this makes him honest as well. Since Xerxes is honest, Yakov's statement is incorrect, but since Yakov is insane, he must have said it honestly. This makes Will's statement true; since we already know Will is dishonest, it follows that he must be insane.

In summary: Will is dishonest and insane, Xerxes is honest and sane, Yakov is honest and insane, and Zhi is honest and sane.