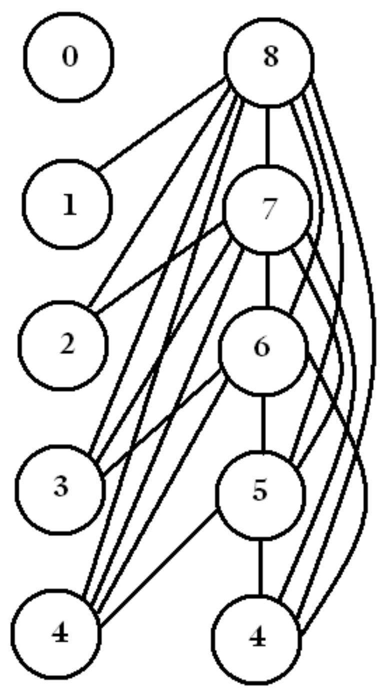
First, observe that everyone shakes between 0 and 8 hands, so for all other 9 people to shake a different number of hands, the number of hands they shook must be $0, 1, \ldots, 8$.

Consider the person who shook 8 hands (call him A_8). He must have shaken hands with everyone but his roommate. But he could not have shaken hands with the person who shook 0 hands (whom we'll call A_0), so A_8 and A_0 must be roommates.

Now, the person who shook 7 hands (A_7, of course) must have shaken hands with everyone but his roommate and A_0, and A_1 must have shaken hands only with A_8. So A_7 and A_1 must be roommates.

By similar reasing, A_6 and A_2 are roommates, and A_5 and A_3 are roommates. This leaves A_4, who must then be your roommate.

Whose hands did you shake? You shook hands with A_8, since he shook hands with everyone but his roommate, and you didn't shake hands with A_0, since he shook hands with no one. By following the reasoning above, you can also conclude that you shook hands with A_7, A_6, and A_5, but not A_1, A_2, or A_3. You didn't shake hands with A_4 since he's your roommate. So, you shook hands with $\boxed{4}$ people. Robin Morillo provided the following illustration of this situation:



If you are allowed to assume that there is a solution in the first place, you can come up with 4 more quickly by recognizing an inherent symmetry in the problem. Suppose that if a pair of non-roommates don't shake hands, then they make awkward eye contact instead. You could restate the problem in terms of awkward eye contact instead of hand shaking, and whatever the answer was for the original problem (call it n), you must get the same answer for the awkward eye contact problem. Since you either shake hands or make awkward eye contact with the 8 people who aren't your roommate, you get $n + n = 8$, so $n = 4$.