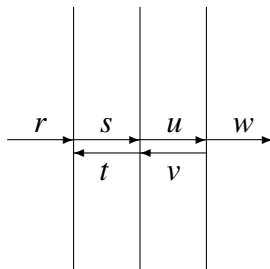


**Technique 1.** Let  $r, s, t, u, v, w$  be the amounts of light labelled in the diagram below. These are relative to the amount of incoming light, so  $t = 0.5$ , for instance, would mean that there is half as much light going from pane 2 to pane 1, as there is light entering pane 1 from the left. Since  $r$  is itself the amount of incoming light, we have  $r = 1$ .



We are trying to find the amount  $w$  of light that comes out.

The light going from pane 1 to pane 2 (which have labelled  $s$ ) will partly come from the 70% of light that hits pane 1 from the left and is transmitted (giving us  $0.7r$ ) and partly come from the 20% of light that hits pane 1 from the right and is reflected (giving us  $0.2t$ ). So we have

$$s = 0.7r + 0.2t$$

We can use similar reasoning to describe the other amounts of light:

$$t = 0.2s + 0.7v$$

$$u = 0.7s + 0.2v$$

$$v = 0.2u$$

$$w = 0.7u$$

Solving this system of equations gives

$$s = 0.7450$$

$$t = 0.2251$$

$$u = 0.5432$$

$$v = 0.1086$$

$$w = 0.3803$$

So, 38.03% of the light gets through. (More exactly,  $343/902$  of the light gets through.)

**Technique 2.** (Most students to solve the problem correctly used something like this.)

We can first analyze the two-pane case, and we get a series for how much light gets through. The first term is how much goes straight through, the second term is how much reflects back and forth once before coming out, the third term is how much reflects back and forth twice before coming out, etc. This yields the series

$$\sum_{k=0}^{\infty} 0.7 \cdot (0.2 \cdot 0.2)^k \cdot 0.7 = \frac{0.49}{1 - 0.04} = 0.51042,$$

using the standard formula for the sum of a geometric series. So the two-pane window transmits 51.042% of the light hitting it. We can similarly analyze how much light is reflected by such a two-pane window, and see that it reflects

$$0.2 + \sum_{k=0}^{\infty} 0.7 \cdot 0.2 \cdot (0.2 \cdot 0.2)^k \cdot 0.7 = 0.2 + \frac{0.098}{1 - 0.04} = 0.30208 = 30.208\%$$

of the light hitting it. So, a two-pane window behaves like a single pane of glass that transmits 51.042% of the light and reflects 30.208% of the light.

Now, we can treat our three-pane window as a two-pane window next to an ordinary pane. This lets us see our system as another kind of two-pane window in which the first “pane” (actually our two-pane window from above) transmits 51% and reflects 30% and the other pane (an ordinary pane) transmits 70% and reflects 20%. Now, we just repeat our two-pane analysis above, but with these new numbers, and we get a window that transmits 38.03% of the light.