Math 225 **Exam 2** **Name:**

8 November 2005 100 Points

"Show enough work to justify your answers."

Write your answers to the first three problems on this page. You may use *Mathematica* as a part of your solution only on the **one** problem indicating it. You may use it to check your answers on any problem.

1. Let $f(x, y, z) = x^2 + xy - yz - z^2$, let P be the point $(2, 3, 5)$, and let $\mathbf{v} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$. (10 points)

   (a) Compute $D_{\mathbf{v}} f(P)$.

   (b) In what direction does f *decrease* most quickly at P? Your answer should be a vector.

2. Consider the surface S in E^3 defined by the equation $xy = (yz + xz)^2$. Verify that $P(4, 1, 2/5)$ is on S and find an equation for the plane tangent to S at P. (10 points)

3. Consider the level curves of a function f as shown. Add ∇f to the picture at the indicated points P, Q, and R. Your vectors do not need to be to scale. (5 points)

READ CAREFULLY: Do five of the remaining problems. You must do at least two of the max/min problems (4, 5, 6). Other than that, you may do any five problems. They are worth 15 points each. If you work on more than five problems, you will get credit for the best five (counting at least two max/min problems). **Please begin each problem on a new sheet of paper.** Put the problem number in the upper right corner of the page. Avoid writing in the upper left corner where the staple will go.

4. Find the global maximum and minimum values of $f(x,y) = x^2 - xy + y^2$ on the closed unit disk $x^2 + y^2 \leq 1$.

5. Let $f(x,y,z) = x^2 + 3\sin(xy) + y^2 + z^2$. Show that the origin is a critical point and determine if f has a local maximum, local minimum, or saddle point there.

6. Find the maximum and minimum values of $f(x,y,z) = xyz$ and their locations on the curve of intersection of the ellipsoid $36x^2+9y^2+4z^2 = 36$ and the plane $6x+3y+2z = 6$. Do all calculus by hand. You may use *Mathematica* to do the algebra. (There is a template in Exam2.nb in the Math225/Foote folder.) If you use *Mathematica*, to get full credit you **must** clearly indicate what you used it for (specifically show the equations solved), what the results are, and what conclusions you draw from it. It is up to you to verify your accuracy in typing expressions into *Mathematica*. Give exact answers if possible. If you have trouble with *Mathematica*, ask for help. *Mathematica* suggestion: Write the gradient equation in vector form and use L instead of λ.

7. Define $f : \mathbf{R^2} \to \mathbf{R}$ by letting $f(x,y)$ be the distance from (x,y) to $(2,-1)$. Assuming (x,y) is some point other than $(2,-1)$, explain why ∇f at the point (x,y) is the unit vector pointing directly away from $(2,-1)$.

8. Suppose the second-order partial derivatives of $f : \mathbf{R^2} \to \mathbf{R}$ are continuous. Prove that $\operatorname{curl}(\nabla f) = 0$.

9. Suppose that $g : \mathbf{R^2} \to \mathbf{R}$ is differentiable, that $w = g(u,v)$, and that $u = x/y$ and $v = z/y$. Show that $x\dfrac{\partial w}{\partial x} + y\dfrac{\partial w}{\partial y} + z\dfrac{\partial w}{\partial z} = 0$.

10. If $f : \mathbf{R^2} \to \mathbf{R}$ is differentiable, compute the expression for ∇f in polar coordinates, that is, ∇f should be expressed as a linear combination of $\mathbf{u}_r$ and $\mathbf{u}_\theta$, and the coefficients should be expressed in terms of r, θ, and f. To do this you will need to first compute the coordinate vector fields $\mathbf{v}_r$ and $\mathbf{v}_\theta$.

11. Suppose $X(t)$ is a curve and $\mathbf{F}$ is a vector field.
    (a) Describe in your own words and/or pictures what it means for $X(t)$ to be a flow curve of $\mathbf{F}$. (5 points)
    (b) Determine if $X(t) = (3\sin t, 2\cos t)$ is a flow curve of $\mathbf{F}(x,y) = \dfrac{2y}{3}\mathbf{i} + \dfrac{3x}{2}\mathbf{j}$. (10 points)

Please put your pages in order before stapling and be sure the problem numbers are in the upper right corners.