Math 225 **Exam 2** **Name:**

11 November 2004 100 Points

1. Write $\int_0^2 \int_{x^2}^{2x} (x^2 + y)\, dy\, dx$ with the order of integration reversed. Do not evaluate. (10 points)

2. Evaluate $\int_{-2}^{2} \int_0^{\sqrt{4-x^2}} \frac{1}{\sqrt{x^2+y^2}}\, dy\, dx$ by changing it to polar coordinates. Do all computations by hand. (10 points)

3. As we know, $\iint_\Omega f(x,y)\, dx\, dy = \iint_\Omega f(r\cos\theta, r\sin\theta) r\, dr\, d\theta$. Show the computation that produces the "area distortion factor" r. (6 points)

4. Let Ω be the solid that is inside both the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 1$. Suppose the mass density of the solid is $\delta(x,y,z) = \sqrt{x^2+y^2+z^2}$. Write two, simplified, ready-to-evaluate integrals that give the mass of the solid, one in cylindrical coordinates and one in spherical coordinates. Do not evaluate. (14 points)

READ CAREFULLY! Do **any four** of the remaining problems. If you work on more than four, you will get credit for the best four. Please do each problem on a separate sheet of paper. *Put the problem number in the* **upper right corner** *and avoid writing anything in the upper left corner where the staple will go.* (15 points each)

5. Let R be the region in $\mathbf{R}^2$ bounded by the parallelogram given by $0 \le 2x - y \le 4$ and $0 \le 3y - 2x \le 8$. (The vertices of this are $(0,0)$, $(3,2)$, $(5,6)$, and $(2,4)$.) Evaluate $\iint_R xy\, dx\, dy$ by changing coordinates to $u = 2x - y$ and $v = 3y - 2x$. After writing a ready-to-evaluate integral, you may evaluate it with *Mathematica*.

6. The graph of the polar curve $r = \cos 2\theta$ for $-\pi/4 \le \theta \le \pi/4$ is shown. (It's one leaf of a four-leaf clover.) Find the area and the x-coordinate of the centroid. After writing the ready-to-evaluate expressions involving integrals, you may evaluate them with *Mathematica*.

7. The curve $\gamma : \mathbf{R} \to \mathbf{R}^2$ given by $\gamma(t) = (t\cos t, t\sin t)$ for $-\pi \le t \le \pi$ is shown. Compute the curvature at the point where the curve goes through the origin. What point is the center of curvature (the center of the osculating circle)? Add the osculating circle to the picture. (You don't need to find the formula for curvature as a function of t, just at the point in question.)

8. Prove that the hyperbola $x^2 - y^2 = 1$ is a smooth one-dimensional manifold.

9. Let M be the set of points in $\mathbf{R}^3$ that satisfy $xy^2 = e^z + \sin(xz)$. Show that the point $(1,-1,0)$ is on M, and that near $(1,-1,0)$ the set M determines z as a differentiable function ϕ of x and y near $(1,-1)$. Find the best affine approximation of ϕ near $(1,-1)$.

10. Find the volume that is inside the sphere $x^2 + y^2 + z^2 = 2$ and outside the cylinder $x^2 + y^2 = 1$. After writing a ready-to-evaluate integral, you may evaluate it with *Mathematica*.

Please put your pages in order before stapling and be sure the problem numbers are in the upper right corners.