Math 225 **Exam 1** **Name:**

18 October 2010 100 Points

"Show enough work to justify your answers."

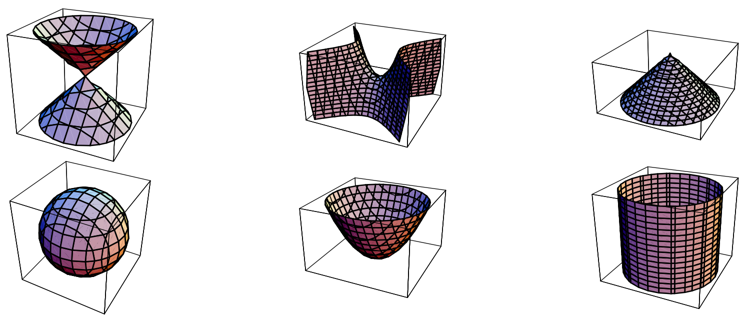
Important: You may use *Mathematica* to help you think on any problem. You may use it as part of your solution only where indicated.

1. Match each graph with its equation. Put the letter of the graph in the blank next to the appropriate equation. Remember that you can determine some features of a surface by setting a variable in its equation to a constant. (10 points)

$z = x^2 + y^2$ ______ $z^2 = x^2 + y^2$ ______ $z = y^2 - x^2$ ______

$r = 3$ ______ $\rho = 2$ ______ $\phi = 3\pi/4$ ______

A B C



D E F

READ CAREFULLY! Do **any six** of the remaining problems. If you work on more than six, you will get credit for the best six. (15 points each)

2. Let $f(x, y) = xy^3 + x^2/y$, and let P be the point $(2, 1)$. Consider the level curve of f passing through P. Find an equation of the line tangent to this level curve at P.

3. Consider the plane passing through $A(2, -1, -1)$, $B(1, 1, 0)$, and $C(2, 0, 1)$. Find the point on this plane that is closest to the point $P(1, -1, 3)$.

4. Let $f(x, y) = x^2y^3$, let $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$, and let P be the point $(2, 1)$. Compute $D_{\mathbf{v}}f(P)$ using the definition of $D_{\mathbf{v}}f(P)$. You may compute it the other way for 10 points partial credit.

5. Let S be the sphere $(x+2)^2+(y-3)^2+z^2=16$. Find the point on S closest to the point $P(2,-1,2)$.

6. Consider $\gamma(t)=(t\cos t, t\sin t)$ for $-\pi \le t \le \pi$.

   (a) Write an integral, simplified and ready to evaluate, that gives the length of the curve. (10 points)

   (b) Use *Mathematica* to evaluate it. Give both the exact answer and a numerical approximation. (If you don't like the exact answer *Mathematica* gives, apply //**TrigToExp** to it.) (5 points)

7. Let $f(X)=f(x,y)=e^{2x}\cos y$. Let $h(X)=h(x,y)$ be the best affine (high school linear) approximation of f near the point $P(0,\pi/4)$. Compute the value of $h\left(-\frac{1}{10},\frac{\pi}{4}+\frac{3}{10}\right)$.

8. Consider the parabola $y=cx^2$ where c is a positive constant. Find a formula for the curvature of the parabola at the origin in terms of c.

9. Do **one** of the following proofs. If you work on both, you will get credit for the best one. Use the back of the page if you need more room.

   (a) Prove the formula $\mathbf{a}=\frac{d^2s}{dt^2}\mathbf{T}+\kappa\left(\frac{ds}{dt}\right)^2\mathbf{N}$.

   (b) Starting with the formula in the previous part, state and prove the formula for curvature.

Selected answers and hints.

2. $5(x-2)+2(y-1)=0$
3. $(5/2, 0, 5/2)$
4. -12
5. $(2/3, 1/3, 4/3)$
6. (b) 12.2198
7. $\sqrt{2}/4$
8. $\kappa = 2c$