Math 225 **Exam 1** **Name:**

29 September 2005 100 Points

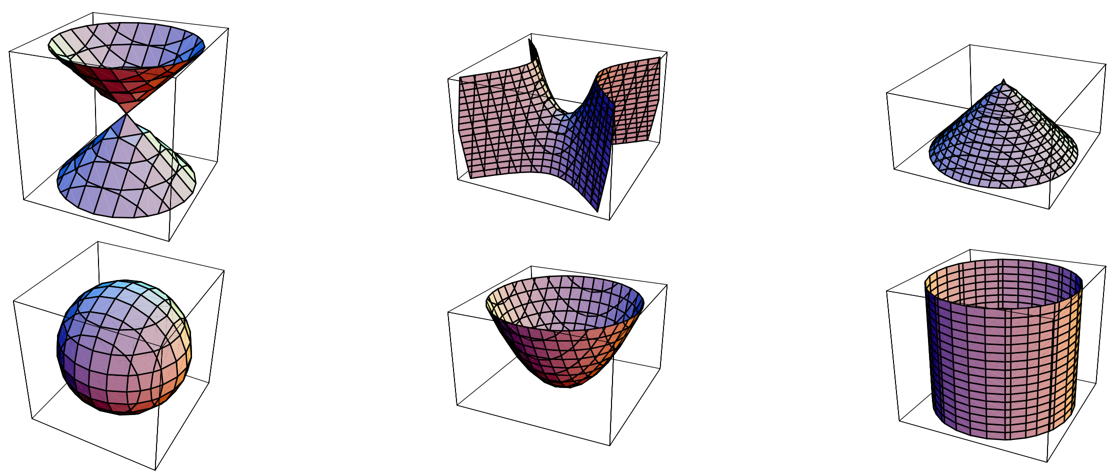
No calculators or *Mathematica*, except as indicated.

"Show enough work to justify your answers."

1. Match each graph with its equation. Put the letter of the graph in the blank next to the appropriate equation. Remember that you can determine some features of a surface by setting a variable in its equation to a constant. (10 points)

$z = x^2 + y^2$ ______ $z = y^2 - x^2$ ______ $z^2 = x^2 + y^2$ ______

$\rho = 1$ ______ $\phi = 3\pi/4$ ______ $r = 1$ ______



READ CAREFULLY! Do **any six** of the remaining problems. If you work on more than six, you will get credit for the best six. Please do each problem on a separate sheet of paper. Put the problem number in the **upper right corner** and avoid writing anything in the upper left corner where the staple will go. (15 points each)

2. Given the points $P(1,1,2)$, $Q(2,-1,-1)$, and $R(2,1,0)$,

   (a) Find an equation of the plane containing them.

   (b) Find the area of the triangle with these points as vertices.

3. Find the point on the plane $3x + 2y - z = 5$ that is closest to the point $P(1,-1,3)$. Suggestion: Use projection or parameterize the line passing through P that is perpendicular to the plane.

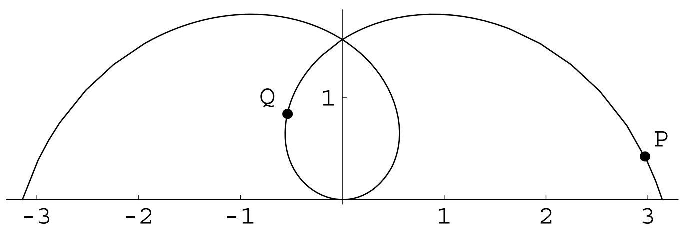
4. Let $f(x, y) = e^{2x} \cos y$. Let P be the point $(0, \pi/4)$.

   (a) Compute $\nabla f(P)$.

   (b) Use the best linear approximation of f near P to approximate the value of $f(-.2, \pi/4 + .3)$.

5. Consider the sphere of radius 2 centered at the point $C(-2, 3, 0)$. Find the point on the sphere closest to the point $P(2, -1, 2)$.

6. The pictured curve is $\gamma(t) = (t \cos t, t \sin t)$ for $-\pi \le t \le \pi$. For parts (a) and (b), draw directly on this picture.



   (a) At the points marked P and Q on the curve, add the vectors $\mathbf{T}$ and $\mathbf{N}$ to the picture. No computation necessary. *Be sure they are to scale.* Suggestion: First determine the direction of the curve.

   (b) Compute the curvature and radius of curvature at $t = 0$ and add the osculating circle at the point where $t = 0$ to the picture. *Be sure the circle is to scale.* Note: To simplify computations you need $\mathbf{v}$ and $\mathbf{a}$ *only* at $t = 0$.

   (c) Write an integral, simplified and ready to evaluate, that gives the length of the curve. Use *Mathematica* to evaluate it. Give both the exact answer and a numerical approximation.

7. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$ does not exist.

8. Prove the formula $\mathbf{a} = \frac{d^2 s}{dt^2}\mathbf{T} + \kappa \left(\frac{ds}{dt}\right)^2 \mathbf{N}$.

9. Starting with the formula in the previous problem, state and prove the formula for curvature.

10. Suppose that $\mathbf{v}(t)$ is a differentiable vector-valued function of t. Suppose that at $t = 5$ the length of $\mathbf{v}(t)$ is a minimum. Prove that $\mathbf{v}(5)$ and $\mathbf{v}'(5)$ are perpendicular.

Please put your pages in order before stapling and be sure the problem numbers are in the upper right corners.

Selected answers and hints.

2. (b) Area: $\sqrt{21}/2$

3. Be careful not to confuse points and vectors. Although it was not required, the only ones who got this completely correct drew a picture. You can solve this either with projection or by parameterizing the line through P perpendicular to the plane. The closest point is $(5/2, 0, 5/2)$.

4. (b) $f(-.2, \frac{\pi}{4} + .3) \approx 0.212132$

5. $(-2/3, 5/3, 2/3)$

6. (b) $\kappa = 2$ (c) $\pi\sqrt{1+\pi^2} + \text{arcsinh}(\pi) \approx 12.2198$. Note: The hyperbolic sine function is given by $\sinh x = (e^x - e^{-x})/2$. Its inverse for $x \geq 0$ can be written as $\text{arcsinh}(x) = \ln(x + \sqrt{1+x^2})$.

10. This is similar to the proof that $\mathbf{v}(t)$ has constant length if and only if $\mathbf{v}(t)$ and $\mathbf{v}'(t)$ are always perpendicular.