Math 225 **Exam 1** **Name:**

5 October 2004 100 Points

1. Find the area of the triangle with vertices $P(1,1,2)$, $Q(2,-1,-1)$, and $R(2,1,0)$. (10 points)

2. Let $f(x,y,z) = ye^{2x}\cos yz$. Compute the partial derivatives of f. (10 points)

3. Let $f(x,y) = \dfrac{x^2}{y+1}$, $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$, and $P = (3,0)$. Compute $D_{\mathbf{v}}f(P)$ using any method you like. (10 points)

4. Let f be as in the previous problem. In what direction does f *decrease* most quickly at the point $Q(-2,1)$? (10 points)

   READ CAREFULLY! Do **any four** of the remaining problems. If you work on more than four, you will get credit for the best four. Please do each problem on a separate sheet of paper. Put the problem number in the **upper right corner** and avoid writing anything in the upper left corner where the staple will go. (15 points each)

5. Find the point on the plane $3x + 2y - z = 5$ that is closest to the point $P(1,-1,3)$. Suggestion: Use projection or parameterize the line passing through P that is perpendicular to the plane.

6. Let $f(x,y,z) = x^2y + xz^2 - y^2/z$. Find an equation of the plane tangent to the level surface of f that passes through $(1,3,-2)$.

7. Consider the sphere of radius 2 centered at the point $C(-2,3,0)$. Find the point on the sphere closest to the point $P(2,-1,2)$.

8. Consider the curve $\gamma : \mathbf{R} \to \mathbf{R}^2$ given by $\gamma(t) = (\frac{2}{\pi}t\cos t, \frac{2}{\pi}t\sin t)$ for $-\pi \le t \le \pi$. Compute the points $\gamma(-\pi)$, $\gamma(-\pi/2)$, $\gamma(0)$, $\gamma(\pi/2)$, and $\gamma(\pi)$. Compute the derivatives $\gamma'(-\pi)$, $\gamma'(-\pi/2)$, $\gamma'(0)$, $\gamma'(\pi/2)$, and $\gamma'(\pi)$. Sketch a reasonably accurate picture of the curve. Label each of the five points with their t-values and show each velocity vector with its tail at the corresponding point on the curve. Choose the scale on your axes appropriately! Suggestion: Plot the points and draw the vectors first. Then use them to draw the curve.

9. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 - y^2}{x^2 + |y|} = 0$.

10. Define $F : \mathbf{R}^2 \to \mathbf{R}^3$ by $F(x,y) = (x^2 + y, xy, x - y)$, or in the book's notation, $F\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^2 + y \\ xy \\ x - y \end{pmatrix}$. Let $P = (2,1)$ and $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$. Compute $D_{\mathbf{v}}F(P)$ in **two** ways: (a) using the definition of directional derivative, and (b) using the derivative matrix.

Selected answers and hints.

3. $D_{\mathbf{v}}f(P) = 21$
4. Note the word "decrease."
6. $10(x-1) + 4(y-3) - \frac{7}{4}(z+2) = 0$
8. If you just plot the points, it's easy to connect them incorrectly. The vectors will indicate how the points should be connected.
9. Break it into two parts.
10. $D_{\mathbf{v}}f(P) = 2\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$