Math 225 **Exam 1** **Name:**

22 October 2002 100 Points

No calculators or *Mathematica*, except as indicated.

"Show enough work to justify your answers."

Read Carefully. Do any **five** problems. If you work on more than five, you will get credit for the best five. Each problem is worth 20 points.

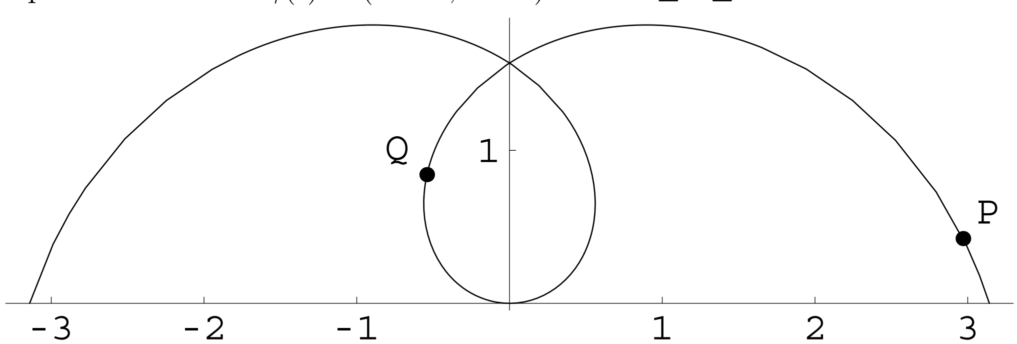
1. Consider plane determined by the points $A(5,0,0)$, $B(6,0,2)$, and $C(5,1,3)$. Show that this plane is parallel to the plane $2x+3y-z=-5$, and find the distance between the two planes.

2. Give a parameterization of the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{9}=1$ that starts $(t=0)$ at $(0,-3)$ and goes counter clockwise. Your work should show that the parameterization satisfies the equation of the ellipse and that it goes in the desired direction.

3. For each function, determine if it is continuous at $(0,0)$. You may use *Mathematica* to help you think, but you may not use it in your answer. Note: The *Mathematica* syntax for $|x|$ is **Abs[x]**.

$$f(x,y)=\begin{cases}\dfrac{xy}{|x|+|y|} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } x=y=0\end{cases} \qquad g(x,y)=\begin{cases}\dfrac{x}{|x|+|y|} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } x=y=0\end{cases}$$

4. The pictured curve is $\gamma(t)=(t\cos t, t\sin t)$ for $-\pi\le t\le\pi$.



   (a) At the points marked P and Q on the curve, add the vectors $\mathbf{T}$ and $\mathbf{N}$ to the picture. No computation necessary. *Be sure they are to scale.* (6 points)

   (b) Compute the curvature and radius of curvature at $t=0$ and add the osculating circle at the point where $t=0$ to the picture. *Be sure the circle is to scale.* Note: To simplify computations you need $\mathbf{v}$ and $\mathbf{a}$ *only* at $t=0$. (7 points)

   (c) Write an integral, simplified and ready to evaluate, that gives the length of the curve. Use *Mathematica* to evaluate it. (7 points)

5. Suppose that $\mathbf{v}(t)$ is a differentiable vector-valued function of t. Suppose that at $t = 5$ the length of $\mathbf{v}(t)$ is a minimum. Prove that $\mathbf{v}(5)$ and $\mathbf{v}'(5)$ are perpendicular.

6. Consider the sphere $x^2 + y^2 + z^2 = 25$. For (x, y, z) in $\mathbf{R^3}$, let $F(x, y, z)$ be the point on the sphere closest to (x, y, z), and let $f(x, y, z)$ be the distance from (x, y, z) to the sphere. Find formulas for $F(x, y, z)$ and $f(x, y, z)$.

7. Consider the maps $S, T : \mathbf{R^2} \to \mathbf{R^3}$ defined by

$$S(\theta, \phi) = (\cos\theta\cos\phi, \sin\theta\cos\phi, \sin\phi) \qquad \text{and}$$
$$T(\theta, \phi) = \big(\cos\theta\,(3 + \cos\phi), \sin\theta\,(3 + \cos\phi), \sin\phi\big).$$

These are identical to the maps we have used that parameterize a sphere and torus. As we have seen, they can also be written as position vectors as

$$S(\theta, \phi) = \mathbf{v}(\theta, \phi) \qquad \text{and} \qquad T(\theta, \phi) = 3\mathbf{u}(\theta) + \mathbf{v}(\theta, \phi), \qquad \text{where}$$
$$\mathbf{u}(\theta) = (\cos\theta)\mathbf{i} + (\sin\theta)\mathbf{j} \qquad \text{and} \qquad \mathbf{v}(\theta, \phi) = (\cos\phi)\mathbf{u}(\theta) + (\sin\phi)\mathbf{k}.$$

Their images are below, *however, note that the pictures are* **not** *the usual Mathematica viewpoint—the x-axis points out of the page and the y-axis points to the right.*

(a) Draw the curve on the sphere with equation $\phi = \pi/4$ and the curve on the torus with equation $\theta = \pi/2$.

(b) Draw the curves on both surfaces corresponding to the equation $\theta = \phi$. Note that the increments $\Delta\theta$ and $\Delta\phi$ used to draw these pictures are equal.

Label all curves with their equations.