Math 223 **Exam 1** **Name:**

18 February 2011 100 Points

You may use a calculator to do arithmetic, but exact answers are expected.

"Show enough work to justify your answers."

1. (a) Let A be the matrix below. Compute A^{-1}. (15 points)

   $A = \begin{pmatrix} 2 & -3 & 6 \\ 1 & -1 & 2 \\ 2 & -2 & 3 \end{pmatrix}$

   (b) Use the result of part (a) to solve the following system, or use any other method. (5 points)

$$\begin{array}{rcrcrcr} 2x & - & 3y & + & 6z & = & 3 \\ x & - & y & + & 2z & = & 1 \\ 2x & - & 2y & + & 3z & = & 0 \end{array}$$

2. A matrix and its reduced echelon form are given.

$$\begin{pmatrix} 2 & -1 & -6 & 5 & 2 \\ 1 & 1 & -3 & 1 & 1 \\ 3 & 2 & -9 & 4 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Given these, what are the solutions of the following systems? Write the solutions in vector form, and as a parameterized line or plane if appropriate. (15 points)

(a)
$$\begin{array}{rcrcrcrcr} 2x_1 & - & x_2 & - & 6x_3 & + & 5x_4 & = & 2 \\ x_1 & + & x_2 & - & 3x_3 & + & x_4 & = & 1 \\ 3x_1 & + & 2x_2 & - & 9x_3 & + & 4x_4 & = & 0 \end{array}$$

(b)
$$\begin{array}{rcrcrcrcrcr} 2x_1 & - & x_2 & - & 6x_3 & + & 5x_4 & + & 2x_5 & = & 0 \\ x_1 & + & x_2 & - & 3x_3 & + & x_4 & + & x_5 & = & 0 \\ 3x_1 & + & 2x_2 & - & 9x_3 & + & 4x_4 & & & = & 0 \end{array}$$

3. Short answers. Do **any four** of the following. If you do more than four, you will get credit for the best four. (20 points)

   (a) Suppose A is a 25×35 matrix and B is a 35×25 matrix. What are the dimensions of AB and BA?

   (b) If A is a matrix, what is meant by the rank of A?

   (c) If $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{w}$ are vectors in the same $\mathbf{R}^n$, what does it mean for $\mathbf{w}$ to be a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$?

   (d) Show that $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ is an eigenvector of $\begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$ and determine the corresponding eigenvalue.

   (e) Explain why a homogeneous system is never inconsistent.

   (f) Explain why a linear system cannot have exactly two solutions.

4. Do **any three** of the following. If you work on more than three, you will get credit for the best three. Continue on the back if you need more room. (45 points)

   (a) Given $\triangle ABC$, let E and F be the midpoints of sides $\overline{AB}$ and $\overline{AC}$, respectively. Use vectors to prove that $\overline{EF}$ is parallel to and half the length of $\overline{BC}$.

   (b) Consider the two lines in $\mathbf{R}^3$ given parametrically as

   $$(x, y, z) = (0, 5, 7) + t(1, -2, -2) \qquad \text{and} \qquad (x, y, z) = (4, 4, 4) + t(2, 3, 1).$$

   Find their point of intersection or prove that they don't intersect.

   (c) Find the point on the plane $3x - 2y + 4z = 0$ that is closest to the point $(8, -3, 7)$.

   (d) Show that $f(x) = 3x + 2$ is not linear in the Math 223 sense.

   (e) Suppose $\mathbf{x}$ and $\mathbf{y}$ are vectors such that $||\mathbf{x}|| = ||\mathbf{y}||$. Prove that $\mathbf{x} + \mathbf{y}$ bisects the angle between $\mathbf{x}$ and $\mathbf{y}$.

Partial answers and hints.

1. Check by multiplication and by plugging in.

2. (a) No solutions.

   (b) $(x_1, x_2, x_3, x_4, x_5) = s(3, 0, 1, 0, 0) + t(-2, 1, 0, 1, 0)$ Note: You can partially check your solution by plugging in. The vectors $(3, 0, 1, 0, 0)$ and $(-2, 1, 0, 1, 0)$ should individually be solutions since the system is homogeneous. What this won't catch is too few free variables.

4. (b) They intersect at $(2, 1, 3)$.

   (c) $(2, 1, -1)$