Math 223 **Exam 3** **Name:**

21 April 2003 100 Points

No *Mathematica*. You may use a calculator to do arithmetic, but exact answers are expected.

"Show enough work to justify your answers."

1. Let $\mathbf{v} = (3, -1, 2)$ and define $L : \mathbb{R}^3 \to \mathbb{R}^3$ by $L(\mathbf{x}) = \text{Proj}_{\mathbf{v}}\mathbf{x}$. This is linear (you don't need to prove this).
   (a) Find the matrix for L. (10 points)
   (b) Find some non-zero vector in the null space of L. Note: Any non-zero vector in the null space will do. You are not being asked to find all of them. You may use the matrix to find one, but you may also guess and verify. (4 points)
   Note: Null space is the same as kernel.
   (c) Let $\mathbf{w} = (2, 1, 1)$. Find the projection of $\mathbf{w}$ into the subspace of $\mathbb{R}^3$ given by $3x - y + 2z = 0$. (Note how this subspace is related to $\mathbf{v}$.) (10 points)

2. Let $A = \left(\begin{smallmatrix} 0 & 2 \\ 2 & 3 \end{smallmatrix}\right)$. (15 points)
   (a) Find the eigenvalues of A.
   (b) For each eigenvalue, find a basis for the corresponding eigenspace.

3. Suppose W is a subspace of $\mathbb{R}^n$. Describe circumstances under which $\text{Proj}_W\mathbf{v}$ is equal to $\mathbf{v}$ for some non-zero vector $\mathbf{v}$. Briefly explain. Hint: Think about the picture for projection. (5 points)

4. Let W be the subspace of $\mathbb{R}^3$ given by the equation $2x + y - 3z = 0$. Find an orthogonal basis for W. Suggestion: Start by finding some basis, and then apply the Gram-Schmidt process. (14 points)

5. Define $L : \mathbb{R}^2 \to \mathbb{R}^2$ by $L(x, y) = (3x - 2y + 4, x - y)$. Determine (with justification) if L is linear or not. (14 points)

6. Consider the basis $\mathcal{B} = \{(2, 1), (-1, 2)\}$ for $\mathbb{R}^2$. Find the coordinates of $\mathbf{v} = (2, 5)$ relative to $\mathcal{B}$. (14 points)

7. Define $D : C^\infty(\mathbb{R}) \to C^\infty(\mathbb{R})$ by $D(f) = f'$. This is linear (you don't need to show it). Show that the function $e^{2x} + e^{-2x}$ is not an eigenfunction of D, but that it is an eigenfunction of D^2, and determine the corresponding eigenvalue for D^2. (14 points)