Math 223 **Exam 2** **Name:**

24 March 2003 100 Points

No *Mathematica.* You may use a calculator to do arithmetic, but exact answers are expected.

"Show enough work to justify your answers."

All problems are worth 12 points except for Problem 2.

1. Find a basis for the null space of the following matrix. $\begin{pmatrix} 4 & -5 & 2 & -2 \\ 1 & -1 & 1 & 0 \\ 3 & -2 & 5 & 2 \\ 1 & 1 & 5 & 4 \end{pmatrix}$

2. What is the rank of the matrix in Problem 1? (4 points)

3. Suppose that A and B are square matrices of the same size, and that $\det A = 2$ and $\det B = -3$. What are $\det(AB)$, $\det(B^2)$, and $\det(A^{-1})$?

4. Let $\mathbf{v}_1 = (3, -2, 1)$, $\mathbf{v}_2 = (2, 3, -4)$, and $\mathbf{w} = (1, 8, -9)$. Determine if $\mathbf{w}$ is in $\text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

5. Given an $n \times n$ matrix, the set of eigenvectors for a particular eigenvalue form a subspace of $\mathbf{R}^n$. This is because these eigenvectors form the null space of a related matrix. Find a basis for the eigenvectors for eigenvalue 6 of the following matrix. Start by finding the eigenvectors. $\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$

6. Show that $f(x) = \cos x$, $g(x) = x \cos x$, and $h(x) = x^2 \cos x$ are linearly independent.

7. Compute the determinant of the following matrix. Remember that the most efficient way is to combine the methods we have seen. $\begin{pmatrix} 0 & -1 & 3 & 4 \\ 1 & 3 & -5 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -2 & 4 & 2 \end{pmatrix}$

8. Suppose that $\mathbf{v}$ is some fixed vector in $\mathbf{R}^{28}$, and let $W = \{\mathbf{x} \in \mathbf{R}^{28} \mid \mathbf{x} \cdot \mathbf{v} = 0\}$, that is, W is the set of all vectors perpendicular to $\mathbf{v}$. Prove that W is a vector space.

9. Give an example (with explanation) of a non-empty subset of $\mathbf{R}^2$ that is not closed under scalar multiplication.