Math 223 **Exam 1** **Name:**

20 February 2003 100 Points

No *Mathematica*. You may use a calculator to do arithmetic, but exact answers are expected.

"Show enough work to justify your answers."

1. Find the solutions of the following systems. (20 points)

   (a)
$$\begin{array}{rcrcrcrcl} x & + & 5y & + & z & + & w & = & 0 \\ x & + & 5y & + & 2z & + & 4w & = & 1 \\ 2x & + & 10y & & & - & 4w & = & 0 \\ & & & & z & + & 3w & = & 1 \end{array}$$

   (b)
$$\begin{array}{rcrcrcrcrcl} x & + & 5y & + & z & + & u & & & = & 0 \\ x & + & 5y & + & 2z & + & 4u & + & v & = & 0 \\ 2x & + & 10y & & & - & 4u & & & = & 0 \\ & & & & z & + & 3u & + & v & = & 0 \end{array}$$

   Note: This system involves the same matrix as the first system. You don't need to do the row reductions again. Just use the reduced matrix you got in the previous part.

2. Let $U = \begin{pmatrix} -5 & 2 & 6 \\ 0 & 1 & -2 \\ 0 & 0 & 4 \end{pmatrix}$ and $L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & -4 & 1 \end{pmatrix}$.

   (a) For what matrix A do these form the LU decomposition? (5 points)

   (b) Find the solution of $AX = B$, where $B = \begin{pmatrix} 6 \\ -5 \\ -12 \end{pmatrix}$. (15 points)

3. Let $A = \begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}$. Write A as a product of elementary matrices. To do this, reduce A to the identity, keeping track of the elementary row operations and the order you use them. A will then be the product (in the appropriate order) of the elementary matrices that reverse this process, turning the identity back into A. Write the product of these elementary matrices, and work out the product to verify that it equals A. (10 points)

4. Given $\mathbf{v} = (3, -2, 1)$ and $\mathbf{w} = (2, 4, 1)$, find the angle between $\mathbf{v}$ and $\mathbf{w}$ in degrees or radians (specify which you are using). (13 points)

5. Let $\mathbf{u} = (0, 2)$, $\mathbf{v} = (3, -1)$, and $\mathbf{w} = (-3, 5)$. Write $\mathbf{w}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$, that is, find scalars a and b such that $a\mathbf{u} + b\mathbf{v} = \mathbf{w}$. (15 points)

6. Show that $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ are eigenvectors of $\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$, and determine the associated eigenvalues. (12 points)

7. Let $V = \{(x, y) \in \mathbf{R}^2 \mid x = 0 \text{ or } y = 0\}$, that is, V is the union of the coordinate axes. Show that V is not a vector space by showing that one of the vector space properties fails to hold. It may help to draw a picture. (10 points)