Math 223 **Exam 2** **Name:**

16 April 2001 100 Points

No *Mathematica*. You may use a calculator to do arithmetic, but exact answers are expected.

"Show enough work to justify your answers."

Exam guidelines.

- This is a take home exam. You may not discuss the contents of the exam with anyone other than me until after it is due and you have turned yours in.
- The exam is open-book and open-note, including the problem sets. You may not refer to any other source.
- The exam is due at 4:30 on Wednesday, 18 April.
- You may use *Mathematica* in your solution of problem 4, but not for any other problem. You may use *Mathematica* to check your solutions for the other problems, but not as part of your solutions. For problem 4, be sure to include all *Mathematica* output that is needed to make your solution complete. Remember to //**Simplify** your answers.
- Begin each numbered problem on a new sheet of paper. Put the problems in order and staple this sheet to the front. Be sure the staple does not obscure the problem numbers or any work.

1. Let $A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & -2 \\ 3 & 1 & 0 \end{pmatrix}$. Compute A^{-1} by hand, showing all steps. Include a check that it is the inverse. (10 points)

2. Suppose that $\mathbf{v} \in \mathbb{R}^2$ and that $\mathbf{v}\cdot(1,-2) = 5$ and $\mathbf{v}\cdot(3,2) = -2$. Find the vector $\mathbf{v}$. (10 points)

3. Define $T\ \mathbb{R}^3 \to \mathbb{R}^3$ by $T(x,y,z) = (2x+3y+2z, 3x+y-2z, -x+2y+4z)$. This is linear (you may assume this).
   (a) Find the matrix of T relative to the standard basis of $\mathbb{R}^3$. (10 points)
   (b) Determine if T is one-to-one. (5 points)
   (c) Determine if T is onto. (5 points)

4. Consider the matrix $A = \begin{pmatrix} 1 & 5 & 1 & 1 & 0 \\ 1 & 5 & 2 & 4 & 1 \\ 2 & 10 & 0 & -4 & 0 \\ 0 & 0 & 1 & 3 & 1 \end{pmatrix}$.

   (a) Find a basis for the column space of A consisting of some of the columns of A. (5 points)

   (b) Find an orthonormal basis for $\mathbb{R}^5$ such that some of the basis vectors form a basis for the subspace of solutions of $A\mathbf{x} = \mathbf{0}$. (15 points)

5. Suppose that A is a fixed $n \times n$ matrix, and that $L : \mathbb{M}(n,n) \to \mathbb{M}(n,n)$ is defined by $L(X) = AX - XA$.

   (a) Prove that L is linear. (10 points)

   (b) Suppose that $n = 2$ and $A = \begin{pmatrix} 1 & 3 \\ -2 & 0 \end{pmatrix}$.

      i. Compute $L\left(\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}\right)$. (5 points)

      ii. Find the matrix for L relative to the basis $\left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right\}$. (10 points)

      iii. Find two independent elements of $\ker L$. (5 points)

6. Suppose that $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are linearly independent vectors in some vector space V and that $L : V \to W$ is linear and one-to-one. Prove that $L(\mathbf{v}_1)$, $L(\mathbf{v}_2)$, and $L(\mathbf{v}_3)$ are linearly independent. (10 points)